Segmentation of 3D Meshes through Spectral Clustering

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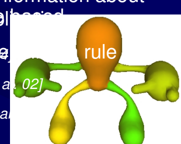
Mesh segmentation

- What ?
- Why ?
- How ?

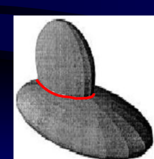
Provides higher-level structural information about



- Geometric morphing [Shlafman et al. 02]
- tion [Li et al. 01]
- Skeleton extraction [Katz & Tal 03]

The minima rule

Definition: all negative minima of the principal curvatures (along their associated lines of curvature) form boundaries between parts. *[Hoffman & Singh 84]*



Decompose 3D shapes at concave creases

Previous work

- Watershed *[Mangan & Whittaker 99]*
  - No need to know how many parts a priori
  - Over segmentation & critical region bypass
- Fuzzy clustering *[Katz & Tal 03]*
  - Intrinsically a "fuzzy" *k*-means clustering
- Space sweeping *[Li et al. 01]*
- Convex decomposition *[Chazelle et al. 94]*

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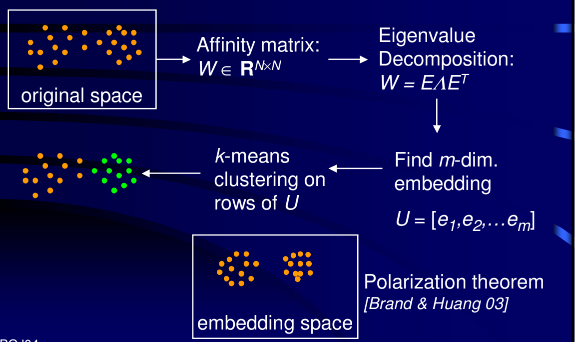
Spectral clustering



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Contents

- Introduction and previous work
- Spectral clustering
- Spectral mesh segmentation algorithm
- Experimental results
- Conclusion + current and future work

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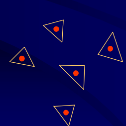
Overview

- On a 2D manifold (triangular face assumed)
- Primitives to cluster — faces
  - Faces provide the simplest and most natural tiling of the surface
  - Segmenting along edges fits well with Minima Rule



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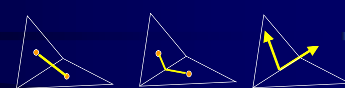
Distance between faces

- Adjacent faces:



  - Distance:

    δ <geodesic distance> + $(1 - \delta)$ <angle distance>

  - Angle distance depends on convexity or concavity
  - Concave angle much more emphasized
- Non-adjacent faces i and j :
  - Distance $D_{i,j}$: shortest distance in (dual) graph

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Affinity matrix & normalization

- Affinity matrix W using exponential kernel

$$\mathbf{W}_{i,j} = e^{-\mathbf{D}_{i,j}/2\sigma^2}$$

- Normalization

$$\mathbf{O} = \mathbf{L}^{-1/2}\mathbf{W}\mathbf{L}^{-1/2}$$

L : diagonal matrix of W's row sums

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m-dimensional embedding

- i-th row of U gives m-dimensional spectral embedding of mesh face i

$$U = [e_1\ e_2 \ldots e_m]$$

  - $e_1\ e_2 \ldots e_m$: m largest eigenvectors of O
- Normalize each embedding coordinate to have unit length

$$\hat{U}_i = U_i / ||U_i||$$

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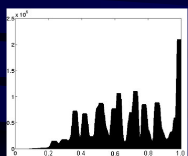
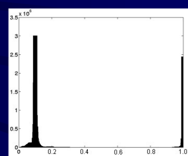
Association matrix

- Computed from spectral embedding $\hat{O} = \hat{U}\hat{U}^T$



Histogram of *W* entries



Histogram of $\hat{O}$ entries

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k-means clustering

- Performed on embedding coordinates based on Euclidean distance
- Initial cluster centers extracted from $\hat{O}$
  - First two centers chosen to be furthest apart
  - Subsequent centers added in a min-max fashion

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Choice of *k* and *m*

- To determine the number of segments *k*
  - Work with original affinity matrix
  - Add representatives in min-max fashion until max affinity between them increases dramatically
  - Not guaranteed to work well in general *[Everitt 01]*
- How many eigenvectors to use?
  - Well-practiced heuristic: same as number of segments

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Execution time (in seconds)

No. Faces	Affinity mat.	Eigen-decompose	Clustering	Total
496	***0.17***	0.16	0.02	0.35
800	***0.51***	0.37	0.03	0.91
1200	***1.7***	0.52	0.09	2.31
1619	***2.7***	1.1	0.17	3.97
2000	***4.76***	1.44	0.29	6.49
4000	***21.35***	6.5	1.72	29.57

Machine configuration: Intel Xeon 2.8GHz, 1GB RAM

Note: Most time spent on constructing affinity matrix

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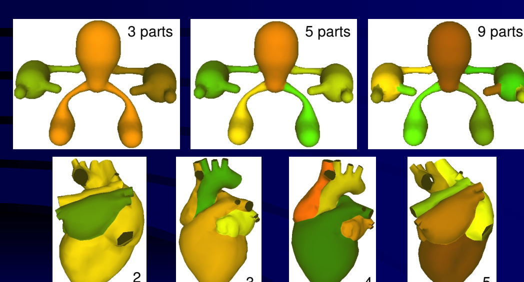
Segmentation results



Hierarchical decomposition via increase in *k*

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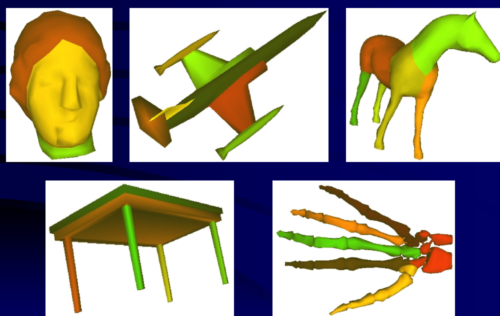
More results



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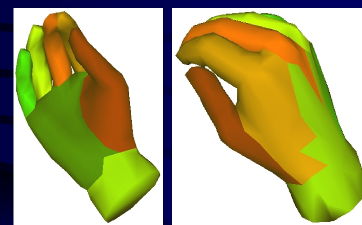
Problematic examples



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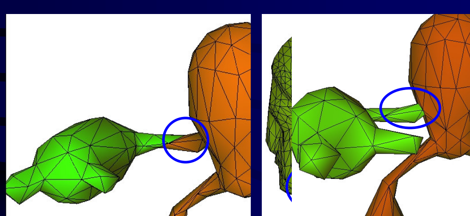
Polarization matters



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Conclusion

- Feasibility and effectiveness of spectral clustering for 3D mesh segmentation
  - Polarization matters
  - Still much to be improved, e.g., timing, parameters
- Ease of implementation
  - Plug in existing eigensolver (Arpack) and standard *k*-means
- Quality
  - Comparable to fuzzy clustering *[Katz & Tal 03]*

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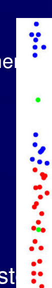
Current and future work

- Sub-sampling algorithm
  - Complexity: $O(|F|^2 log|F|) \rightarrow O(s|F|log|F|)$, whe s number of samples and is really a constant
- Hierarchical bisection
  - Avoids choosing *k*
  - Improves segmentation quality
- Boundary smoothing via morphology
- Replace *k*-means by more advanced clust g



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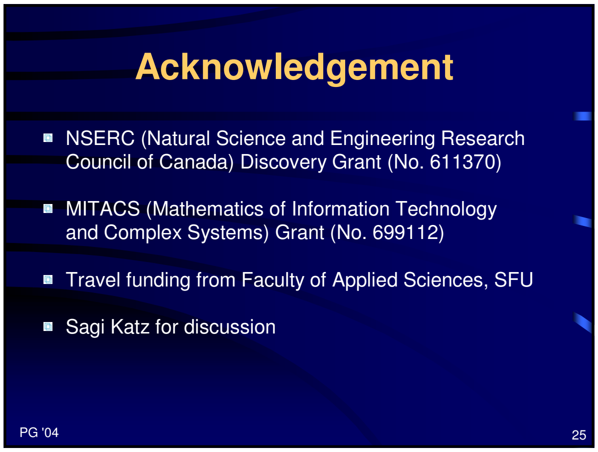
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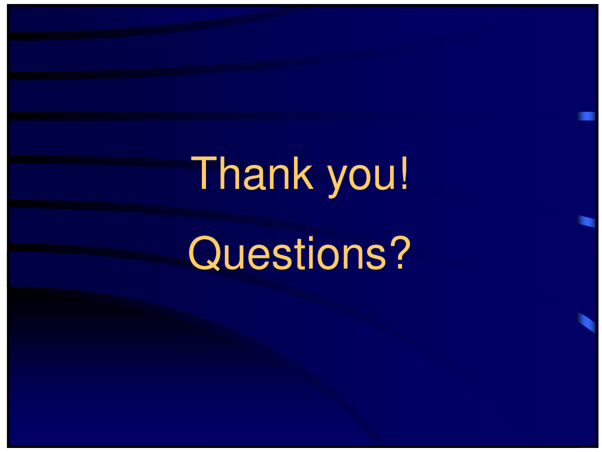
Thank you!
Questions?