

# Mathematical Surface Representations for Conceptual Design

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# Conceptual design

- What is conceptual design?

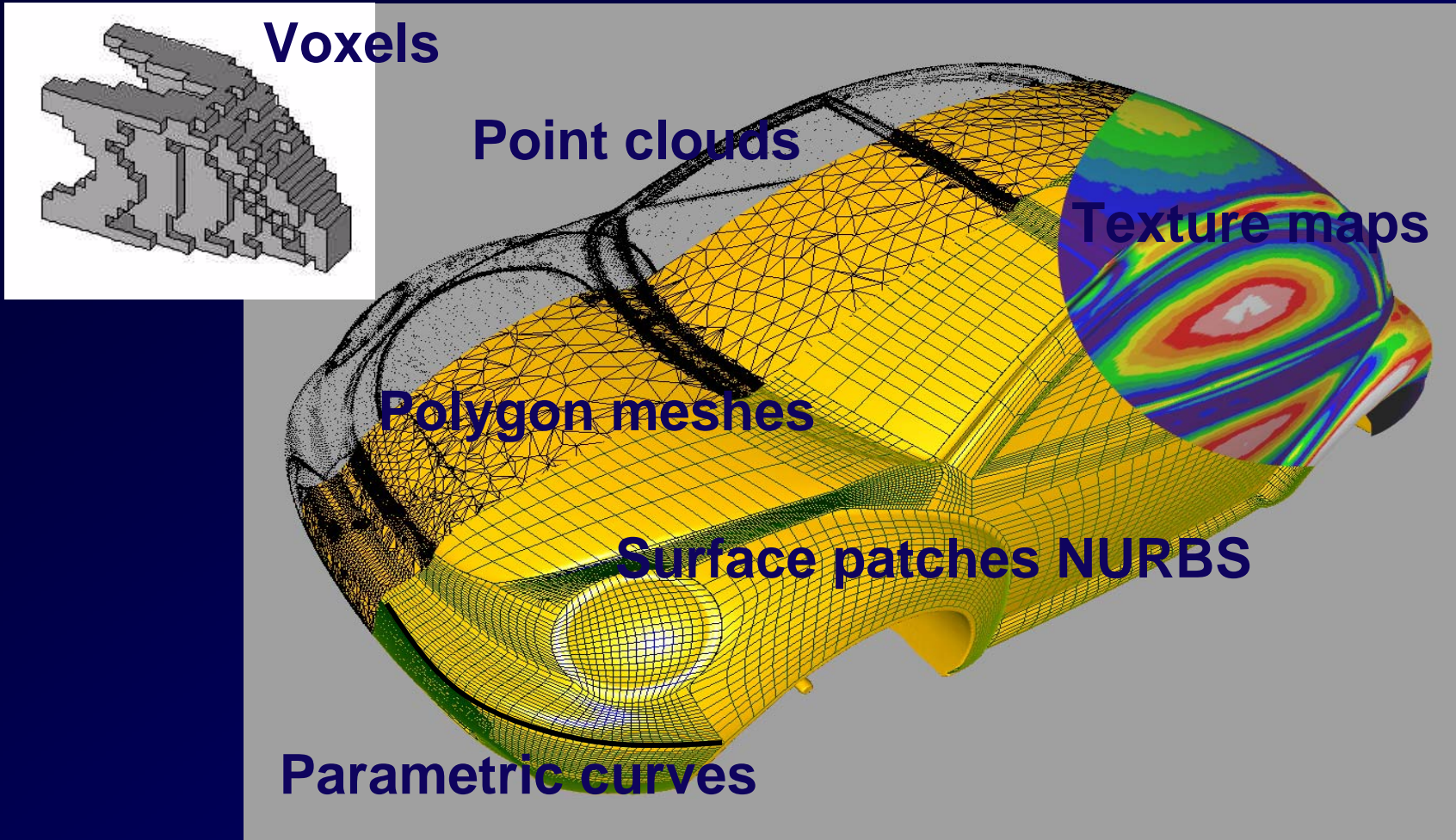
# Conceptual design desirables

- Abstraction from underlying surface math
- Invite creative exploration
- Allow for precision and constraints
- Workflow mimics traditional design media
- Intuitive and interactive



# Surface representations

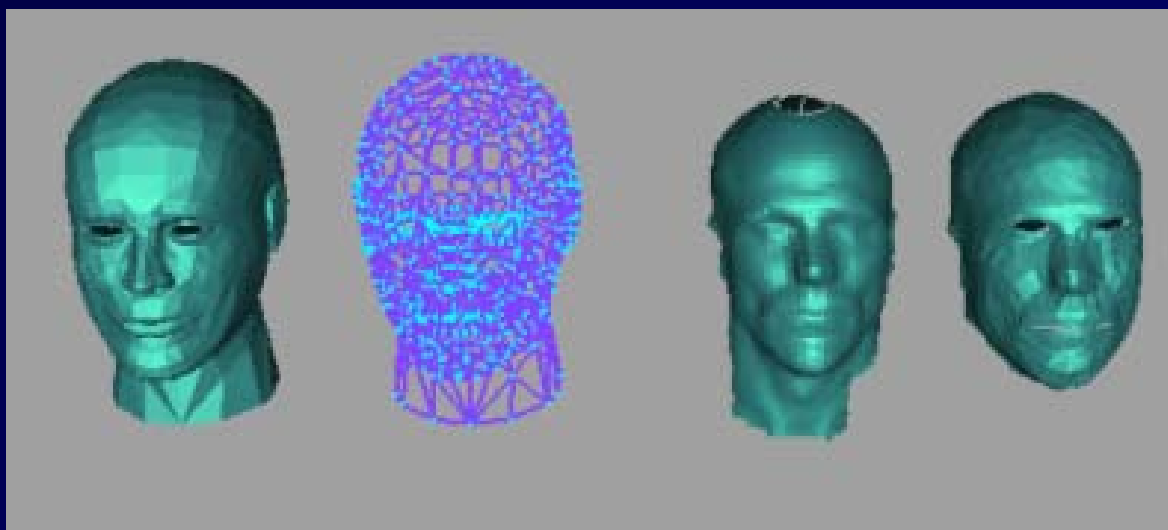
How do we represent an object mathematically?



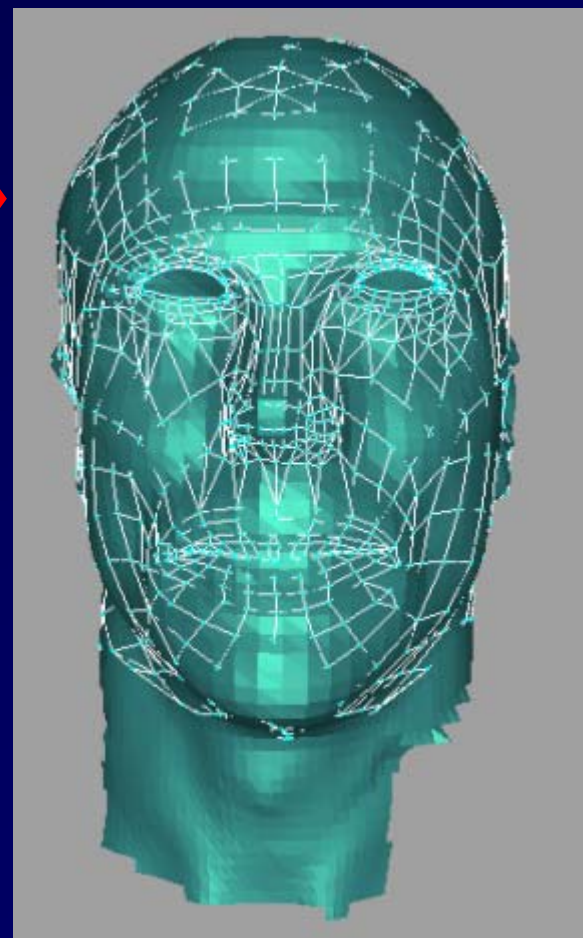
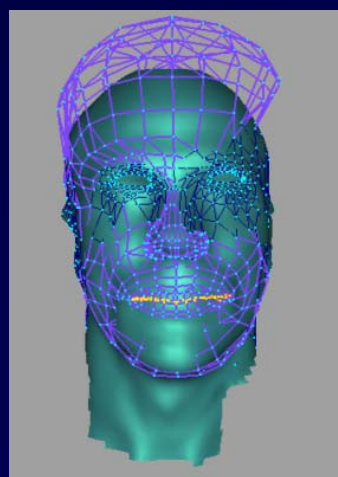
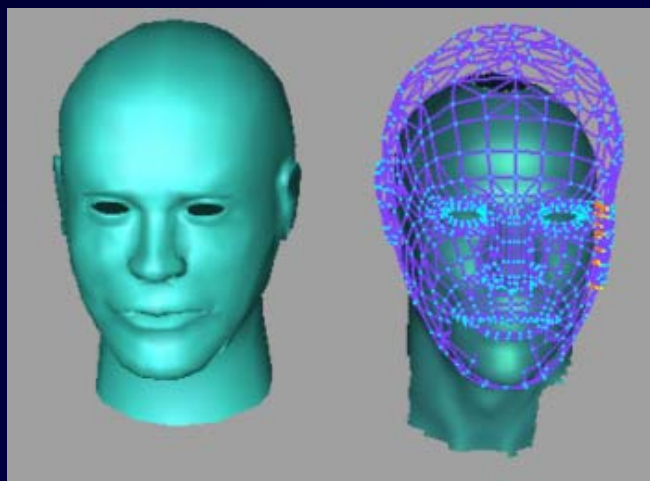
# Feature based retargeting of parameterized geometry

Problem:

Given geometry  $M_1$  with a global parameterization  $P_1$ , retarget  $M_1$  to unstructured geometry  $M_2$  while minimizing the difference in some *feature* of geometry between corresponding parameter values of  $M_1$  and  $M_2$ .



# Feature based retargeting of parameterized geometry

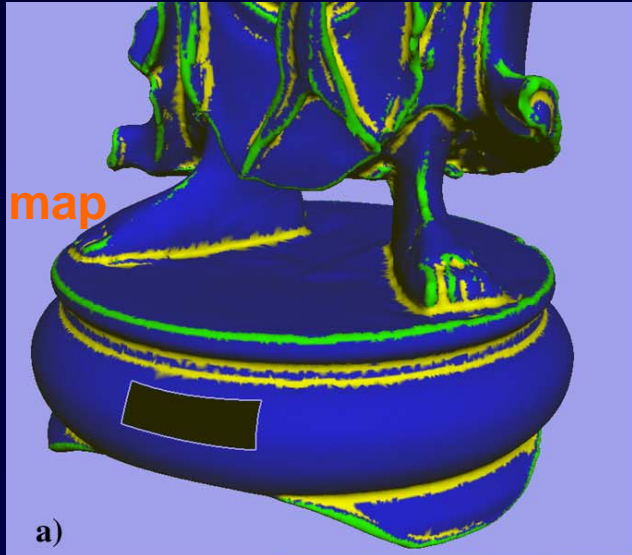


**Feature definition & alignment**

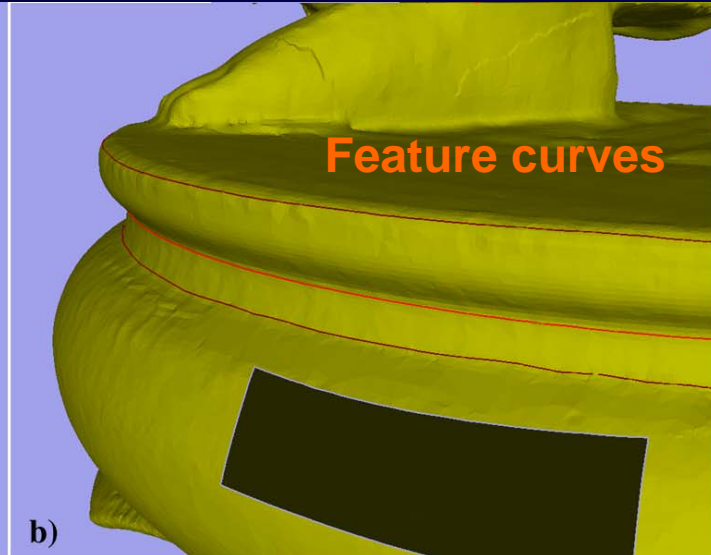
**Parametric relaxation**

# Feature definition and alignment

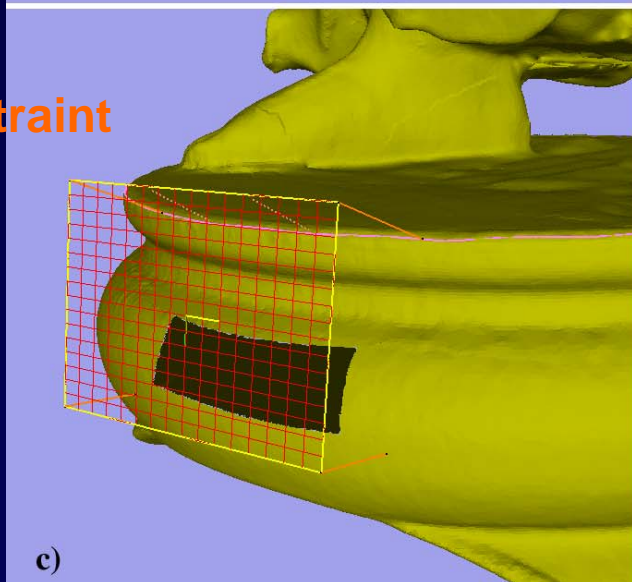
Curvature map



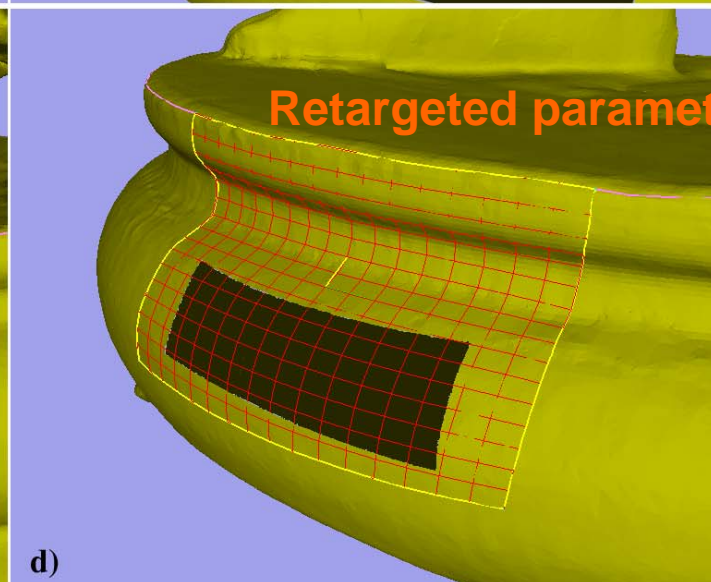
Feature curves



Curve constraint



Retargeted parametric patch



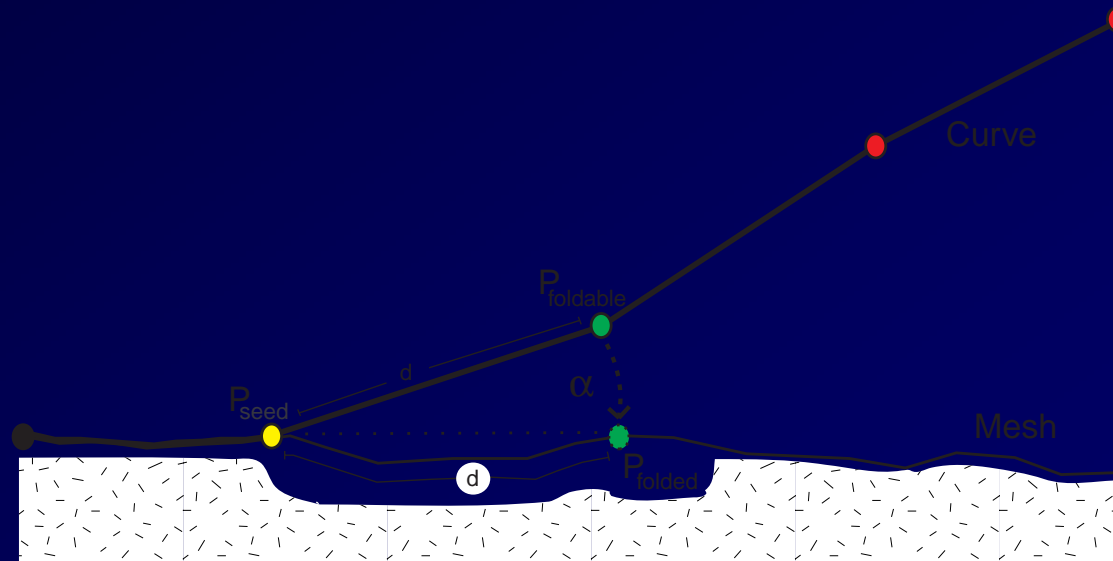
# Parametric relaxation

- Geometry being retargeted needs a hybrid formulation: since real geometric data has noise and holes that parameterization must be insensitive towards.
- A point  $P$  from  $M_1$  has both a 3D position  $P_s$  (space point) and a 2D value  $P_f$  (face point) in some local parameterization of the target geometry  $M_2$ . (in the case of meshes we use barycentric coordinates).
- Retargeting is formulated as an iterative constraint optimization using energy functions.
- Our energy terms include:
  - 3D thin-plate energy (curvature continuity in unconstrained regions).
  - 2D surface energy (minimize internal distortion of parameterization).
  - Feature energy (attraction of points to geometric features).
  - Folding energy (controls the transition of unconstrained space points to constrained face points).

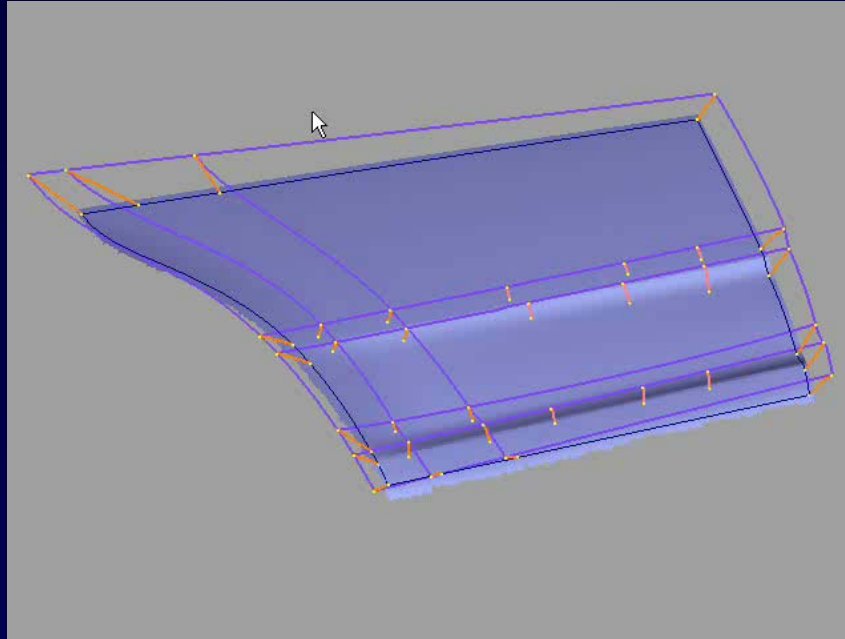


# Folding Energy

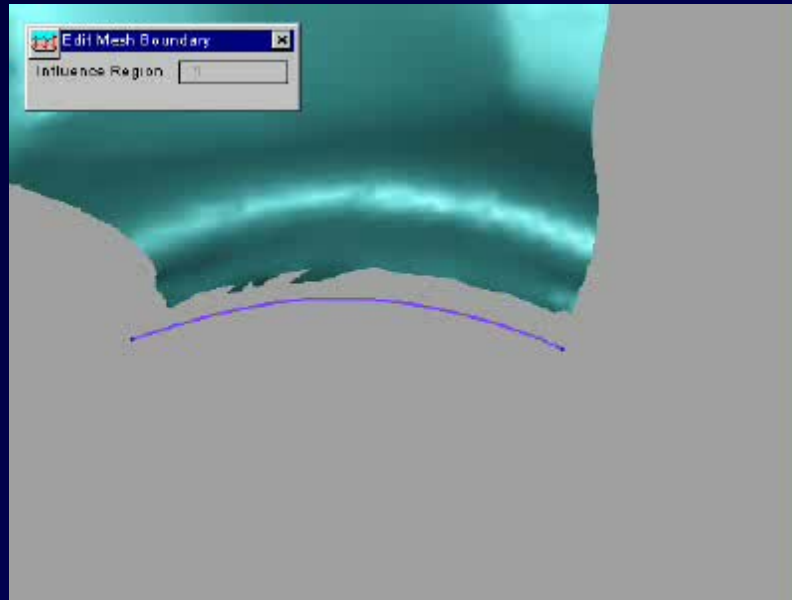
- $P_{foldable}$  is an unconstrained space point connected to  $P_{seed}$  a constrained face point.
- Folding energy is a function of the angle  $\alpha$ , drawing points from space towards the target geometry.
- Simulated annealing adaptively changes the energy value based on the number of points folded in a prior iteration.



# Results



# Applications (fixing geometry)



# Suggestive interface for image guided 3D sketching

- Gestures
- Image-guided drawing
- Suggestions

# Suggestive interface for image guided 3D sketching

A Suggestive Interface for Image Guided  
3D Sketching

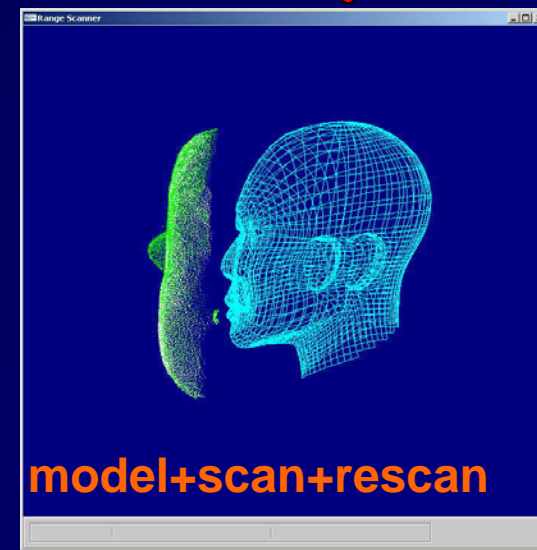
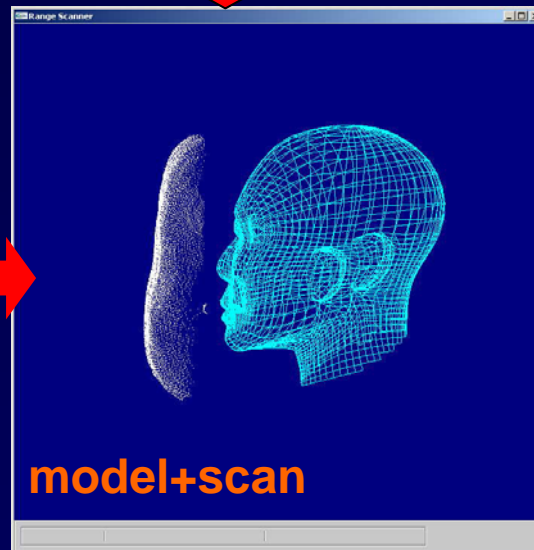
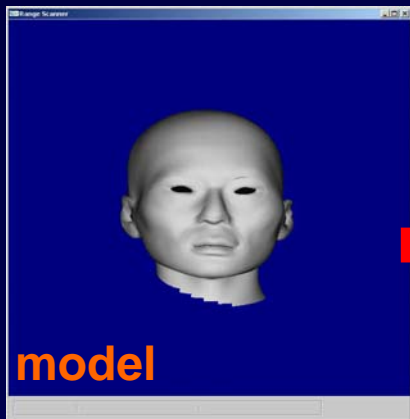
# Sculpting digital models

How can one use physical media like clay to deform mathematically represented geometry?



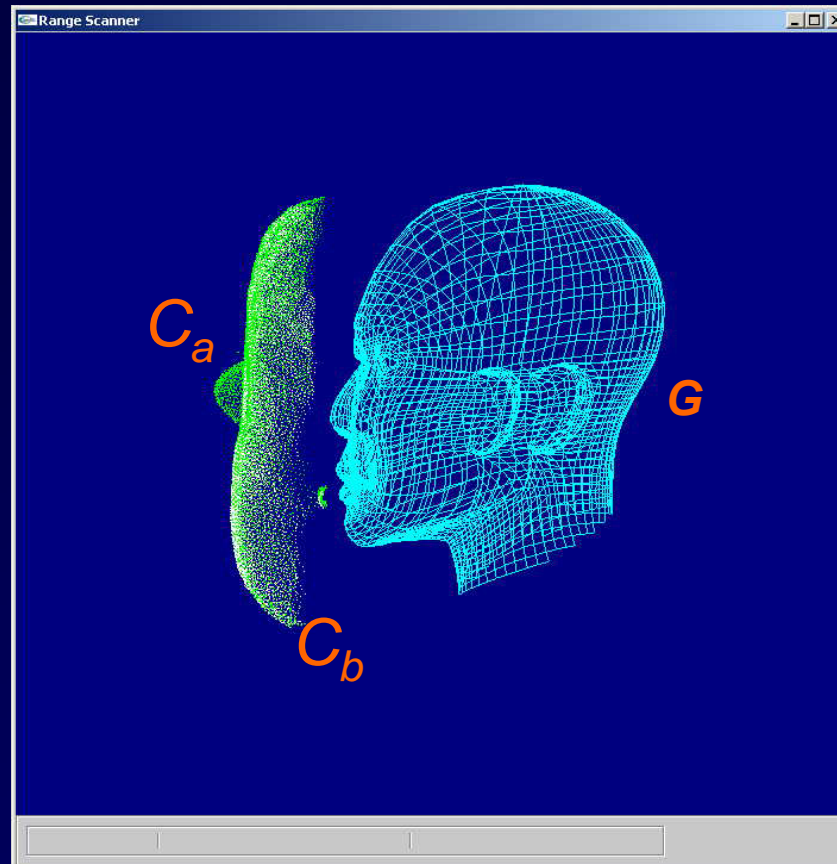
# Sculpting digital models

Workflow:



# Sculpting digital models

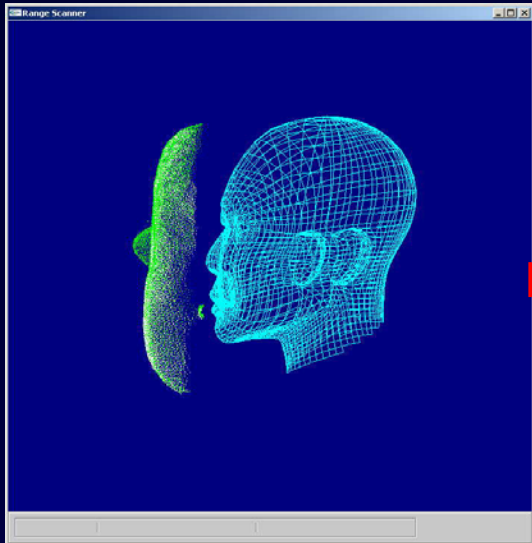
Problem: Given two sample 3D point data sets  $C_b, C_a$  and a piece of 3D geometry  $G$ , deform  $G$  to reflect the difference between  $C_b$  and  $C_a$ .



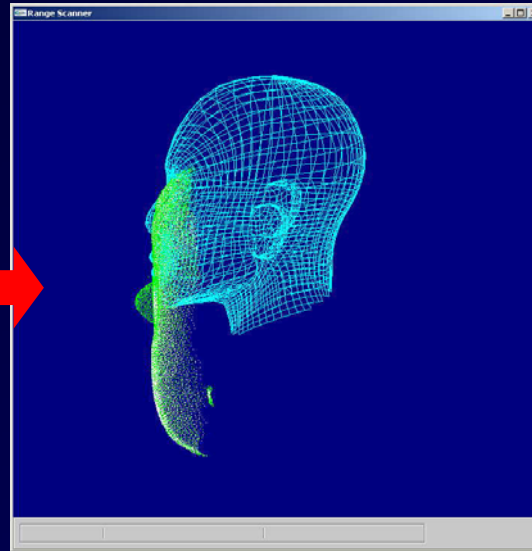


# Sculpting digital models

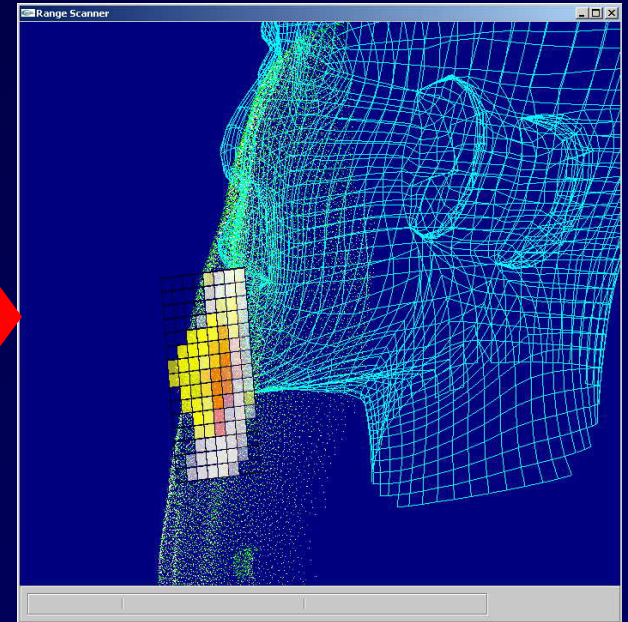
Approach:



Align  $C_b, C_a$



Align  $G$  with  $C_b, C_a$



Voxelize  $C_b, C_a$

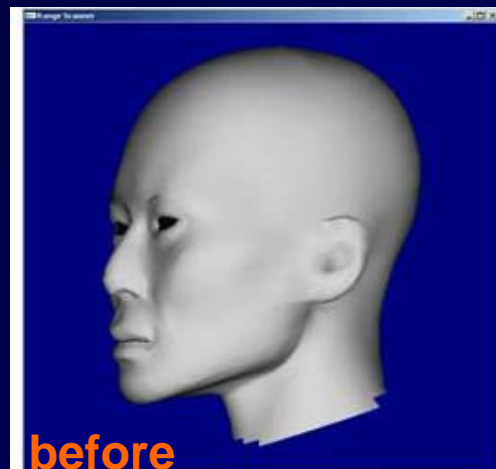
Map  $f: C_b - (C_a \cap C_b) \rightarrow C_a - (C_a \cap C_b)$

Map  $g: G \rightarrow C_b$

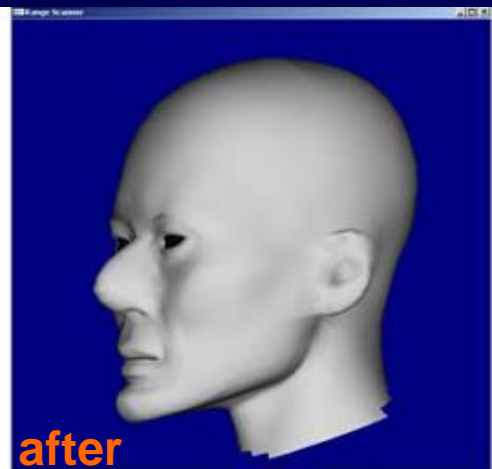


Correct topology of  $G$ .

Deform points of  $G$  using maps



before



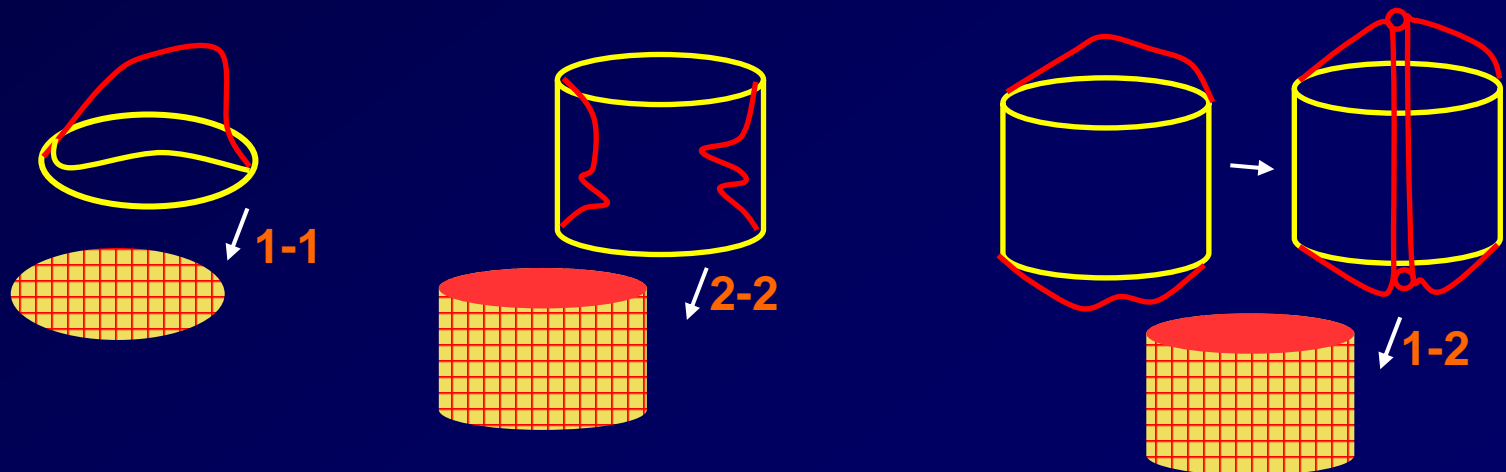
after

# Algorithm

- **Voxelize  $C_b, C_a$ .**
  - Compute centroid and a weight value for each voxel based on the distribution and number of scan point samples within voxel grid.

# Algorithm

- **Voxelize  $C_b, C_a$ .**
- **Map  $f: C_b - (C_a \cap C_b) \rightarrow C_a - (C_a \cap C_b)$ .**
  - Filter the voxel representations of  $C_b - (C_a \cap C_b)$  and  $C_a - (C_a \cap C_b)$  to reduce the number of connected components.
  - Match before and after components based on shared boundaries.
  - 1-1 boundary  $\Rightarrow$  parametrically map both to surface of a disk.
  - 2-2 boundary  $\Rightarrow$  parametrically map both to surface of a cylinder.
  - 1-2 boundary  $\Rightarrow$  cut a hole in the center of 2 disks, connect the holes and map both to surface of a cylinder.



# Algorithm

- **Voxelize  $C_b, C_a$ .**
- **Map  $f: C_b - (C_a \cap C_b) \rightarrow C_a - (C_a \cap C_b)$ .**
- **Map  $g: G \rightarrow C_b$** 
  - **Map any point  $p$  on the geometry to a set of proximal voxels of  $C_b$**

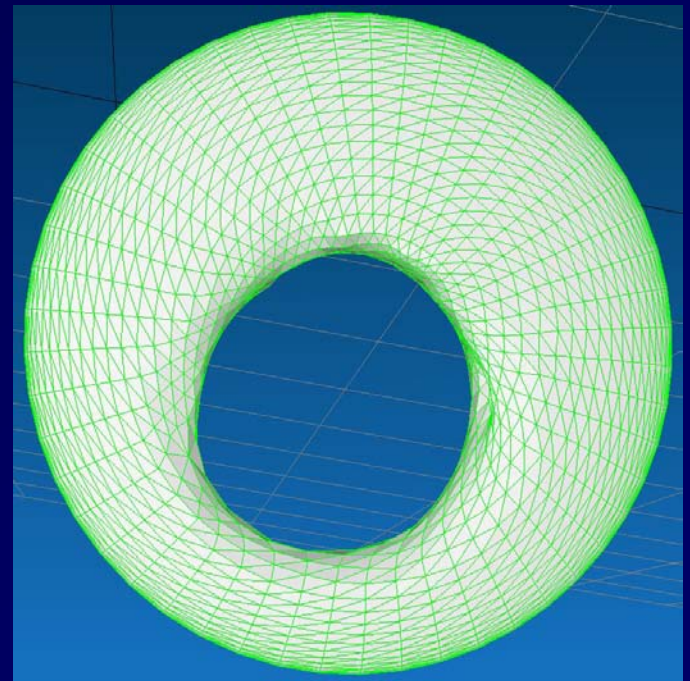
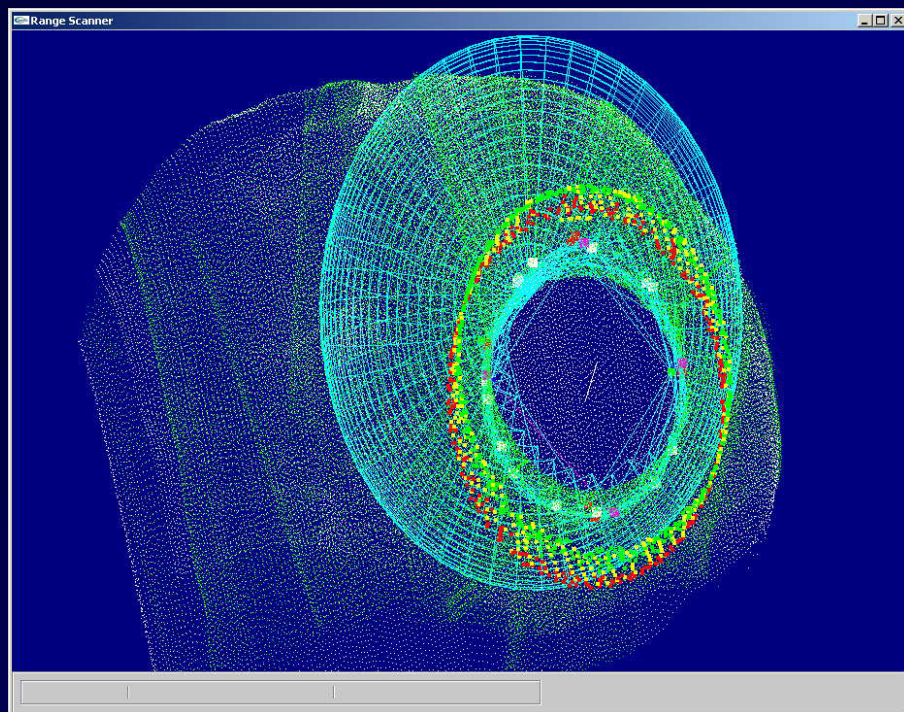
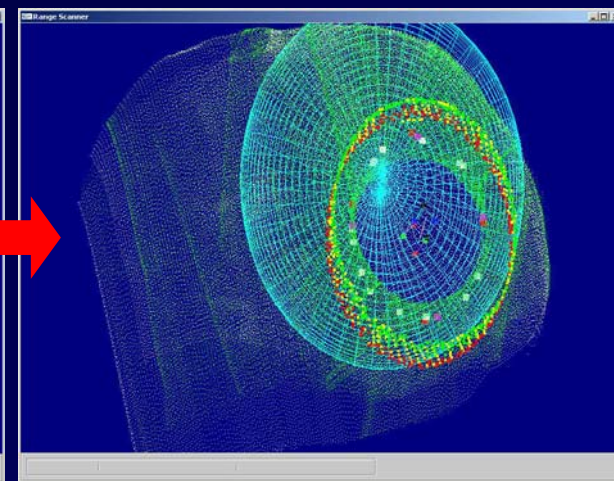
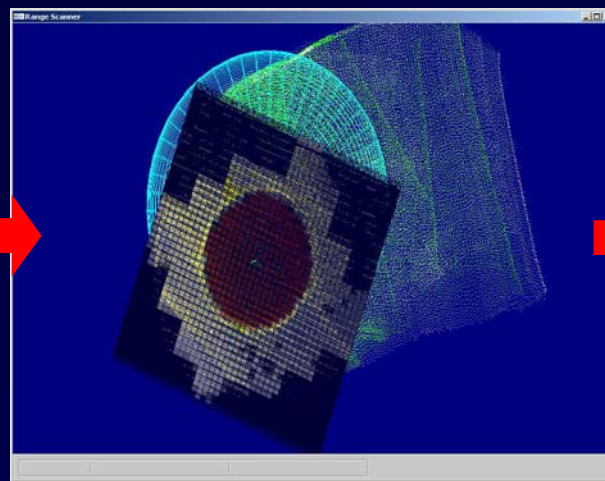
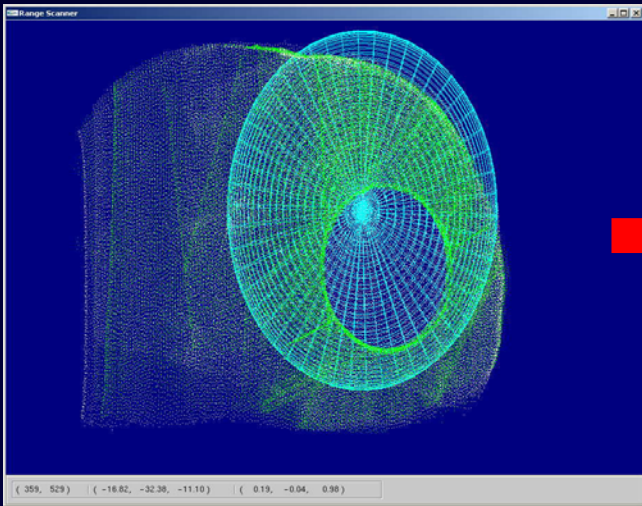
# Algorithm

- **Voxellize  $C_b, C_a$ .**
- **Map  $f: C_b - (C_a \cap C_b) \rightarrow C_a - (C_a \cap C_b)$ .**
- **Map  $g: G \rightarrow C_b$**
- **Correct topology of  $G$ .**
  - *If 1-2 boundary mapping intersect line of cut with  $G$  and join pairs of intersection to change the genus of  $G$ .*

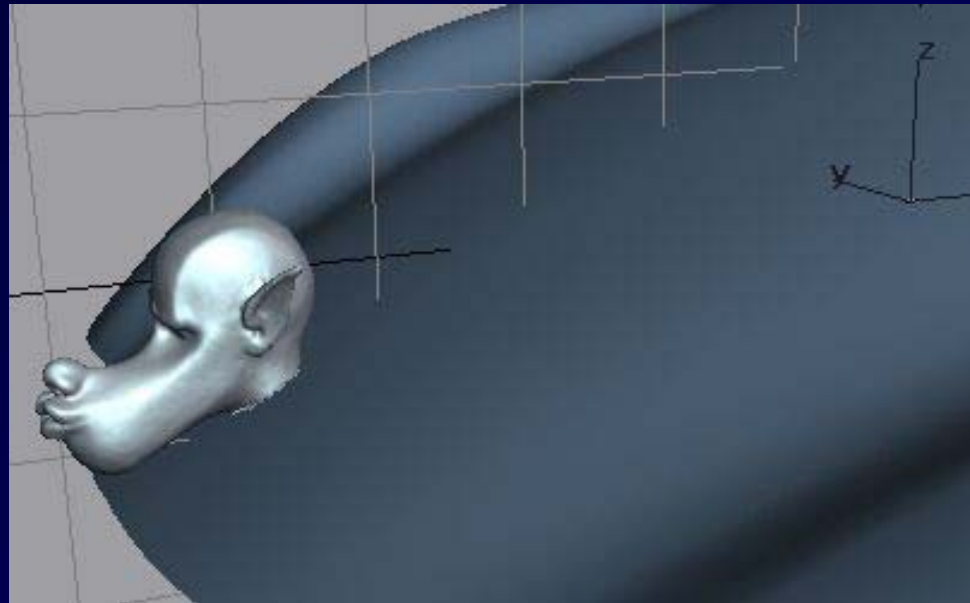
# Algorithm

- **Voxelize  $C_b, C_a$ .**
- **Map  $f: C_b - (C_a \cap C_b) \rightarrow C_a - (C_a \cap C_b)$ .**
- **Map  $g: G \rightarrow C_b$**
- **Correct topology of  $G$ .**
- **Deform points  $p \in G$  to  $p += \alpha^*(f(g(p)) - g(p))$ .**

# Results



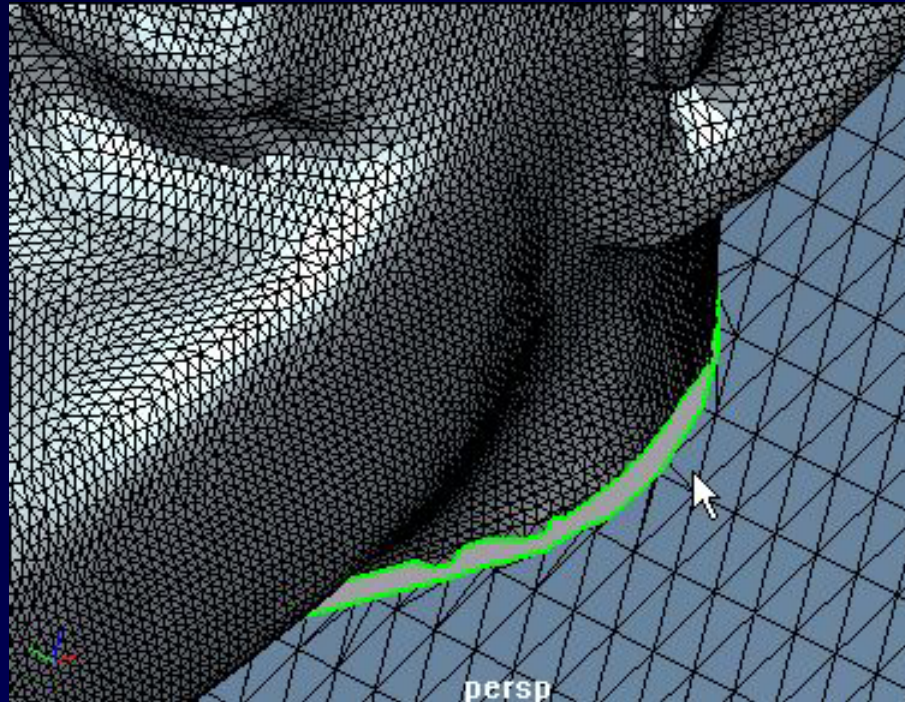
# Cut and Paste



**Hood ornament to be pasted:  
making a Wolf out of a Jaguar**

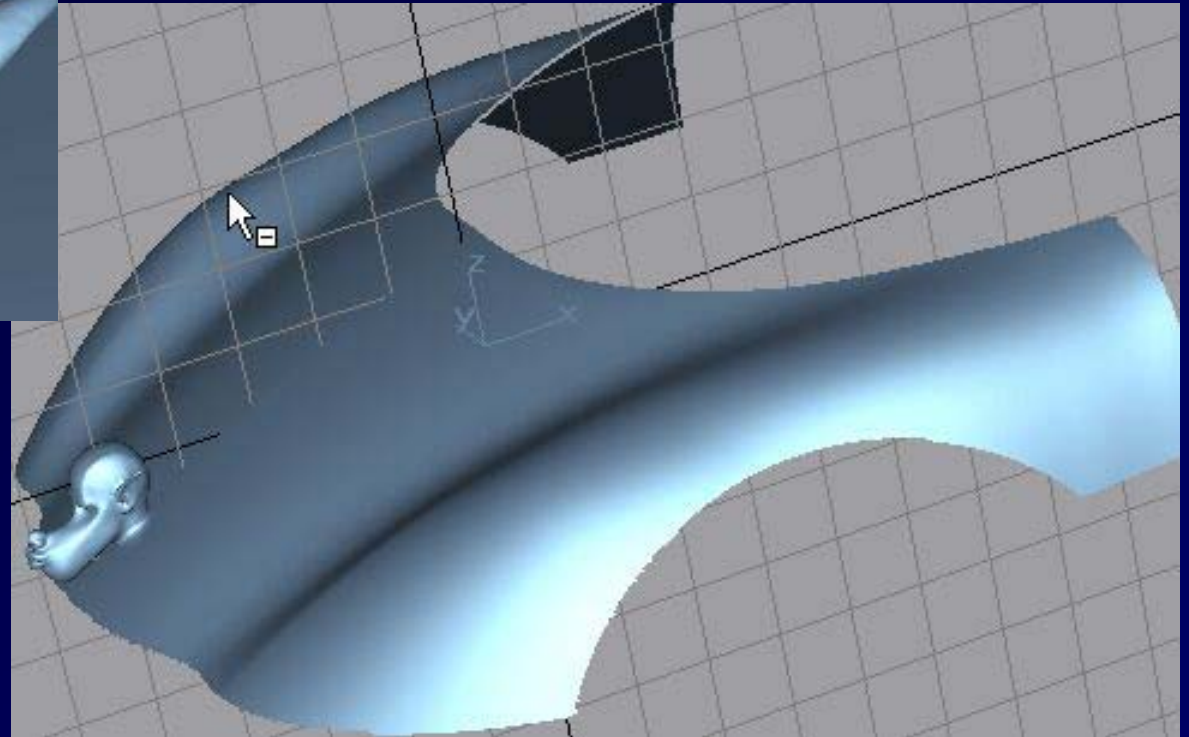
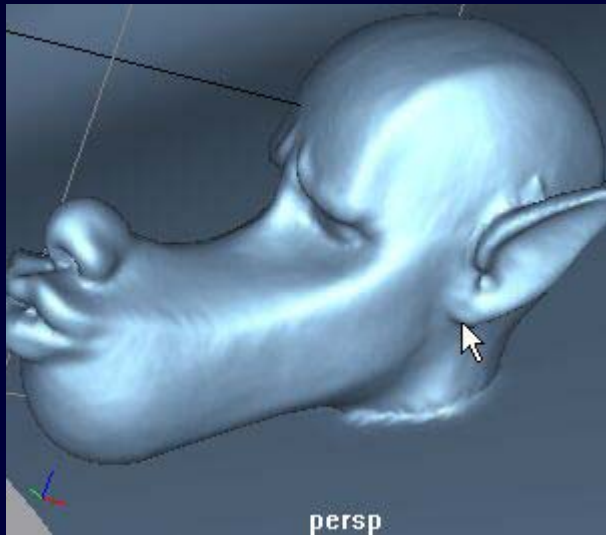


# Cut and Paste



Holes prior to being stitched

# Cut and Paste



**Pasted hood ornament**