CSC418: Computer Graphics

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Today’s Topics

1. Texture mapping
2. More Ray Tracing

Some slides and figures courtesy of Wolfgang Hürst, Patricio Simari
Some figures courtesy of Peter Shirley,
“Fundamentals of Computer Graphics”, 3rd Ed.

Showtime

https://www.youtube.com/watch?v=frLwRLS_ZR0

But First … Logistical Things

- Assignment 3 available on BBS (coming soon to website)

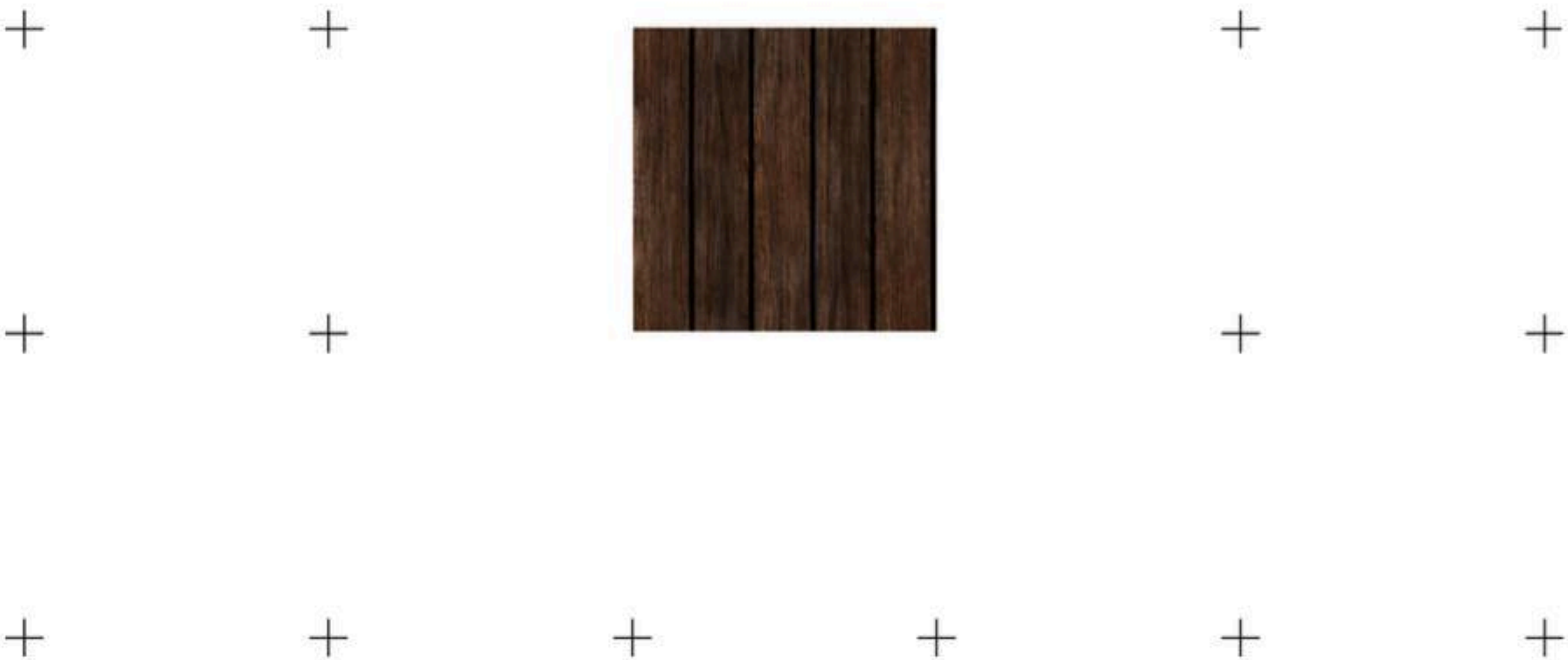
Topic 1:

Texture Mapping

- Motivation
- Sources of texture
- Texture coordinates
- {Bump, MIP, displacement, environmental} mapping

Motivation

- Adding lots of detail to our models to realistically depict skin, grass, bark, stone, etc., would increase rendering times dramatically, even for hardware-supported projective methods.



Motivation

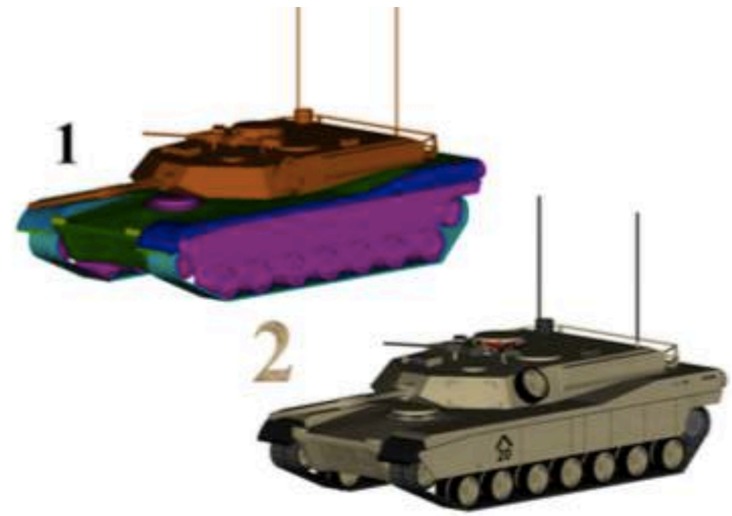
- Adding lots of detail to our models to realistically depict skin, grass, bark, stone, etc., would increase rendering times dramatically, even for hardware-supported projective methods.

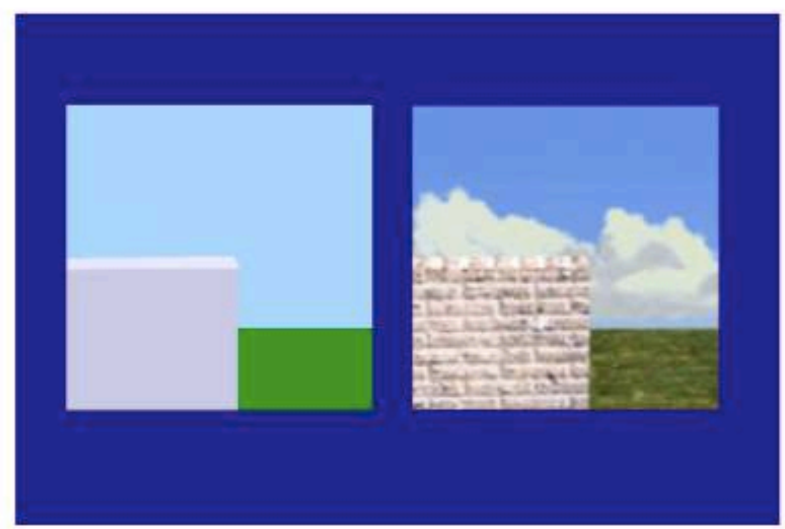
Motivation

Basic idea of texture mapping:

Instead of calculating color, shade, light, etc. for each pixel we just paste images to our objects in order to create the illusion of realism

Different approaches exist (e.g. tiling; cf. previous slide)





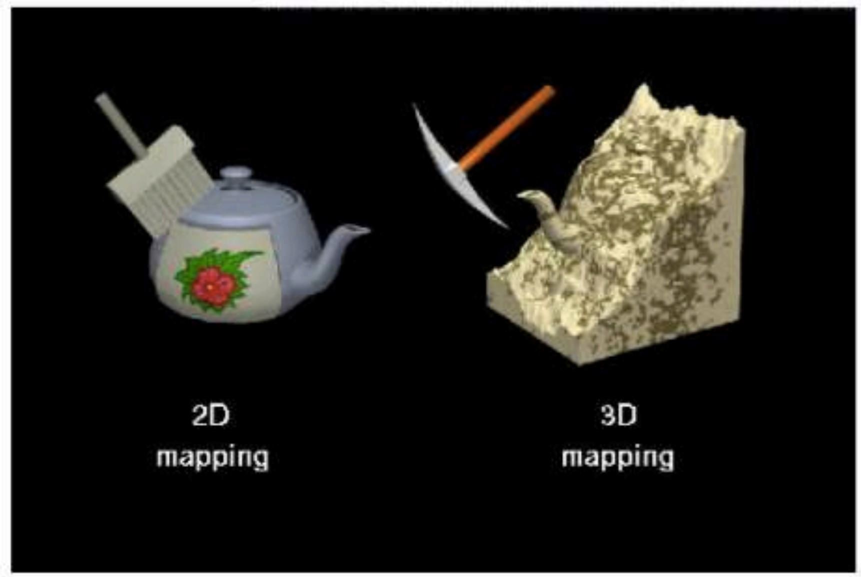
Motivation

In general, we distinguish between 2D and 3D texture mapping:

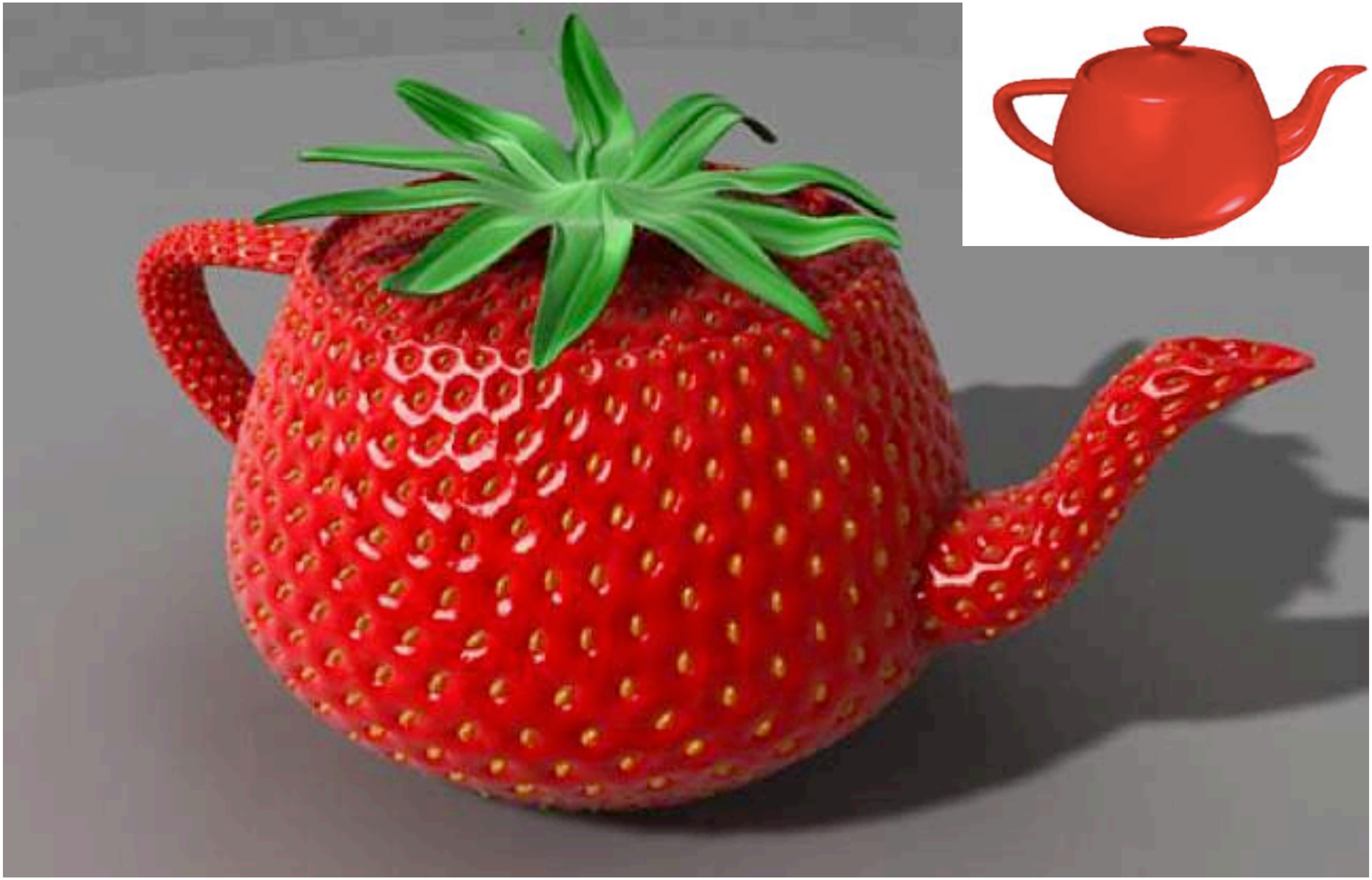
2D mapping (aka *image textures*): paste an image onto the object

3D mapping (aka *solid* or *volume textures*): create a 3D texture and "carve" the object

3D Object



2D texture $\longleftrightarrow$ 3D texture



Topic 1:

Texture Mapping

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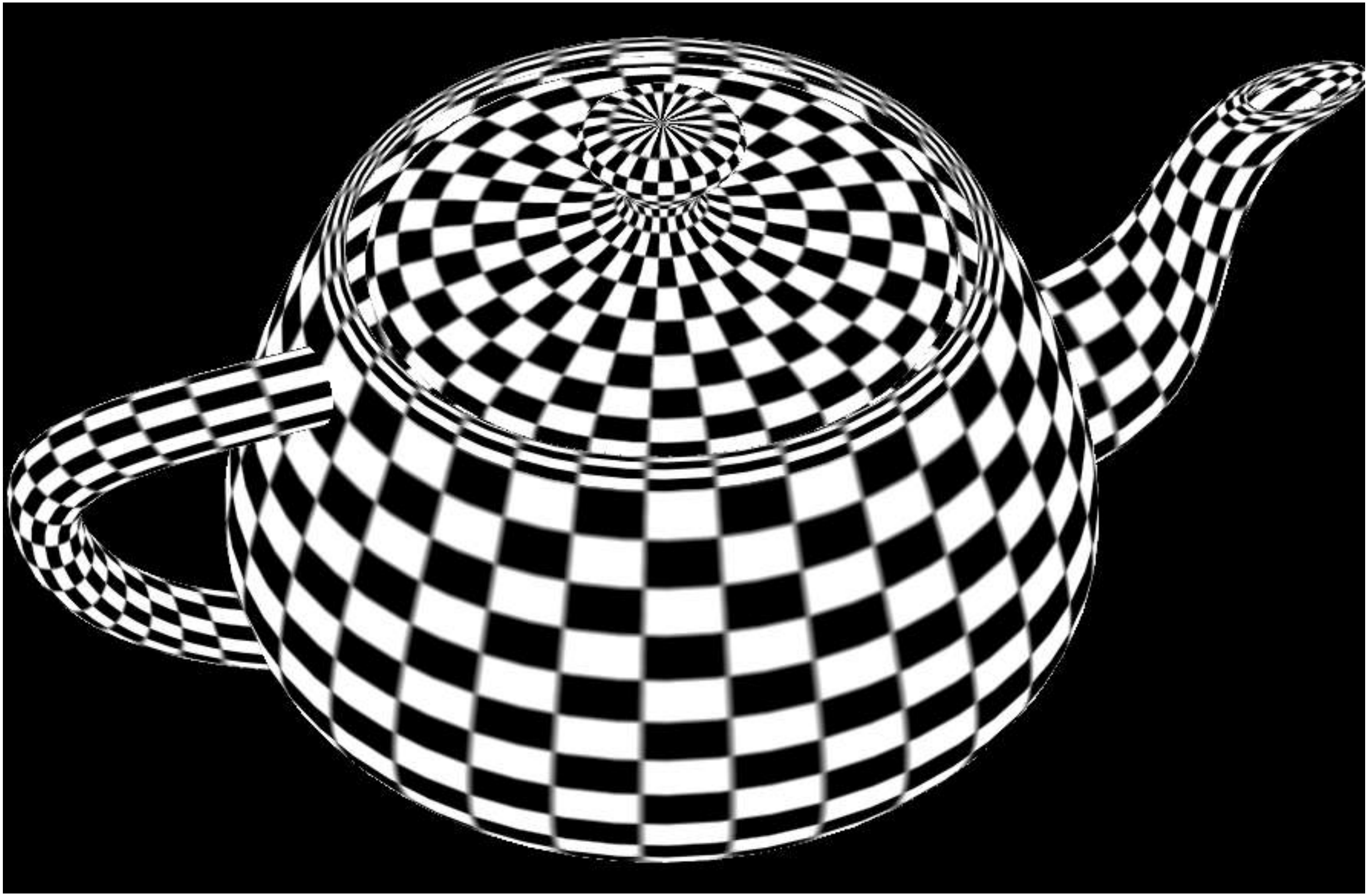
- Sources of texture
- Texture coordinates
- {Bump, MIP, displacement, environmental} mapping

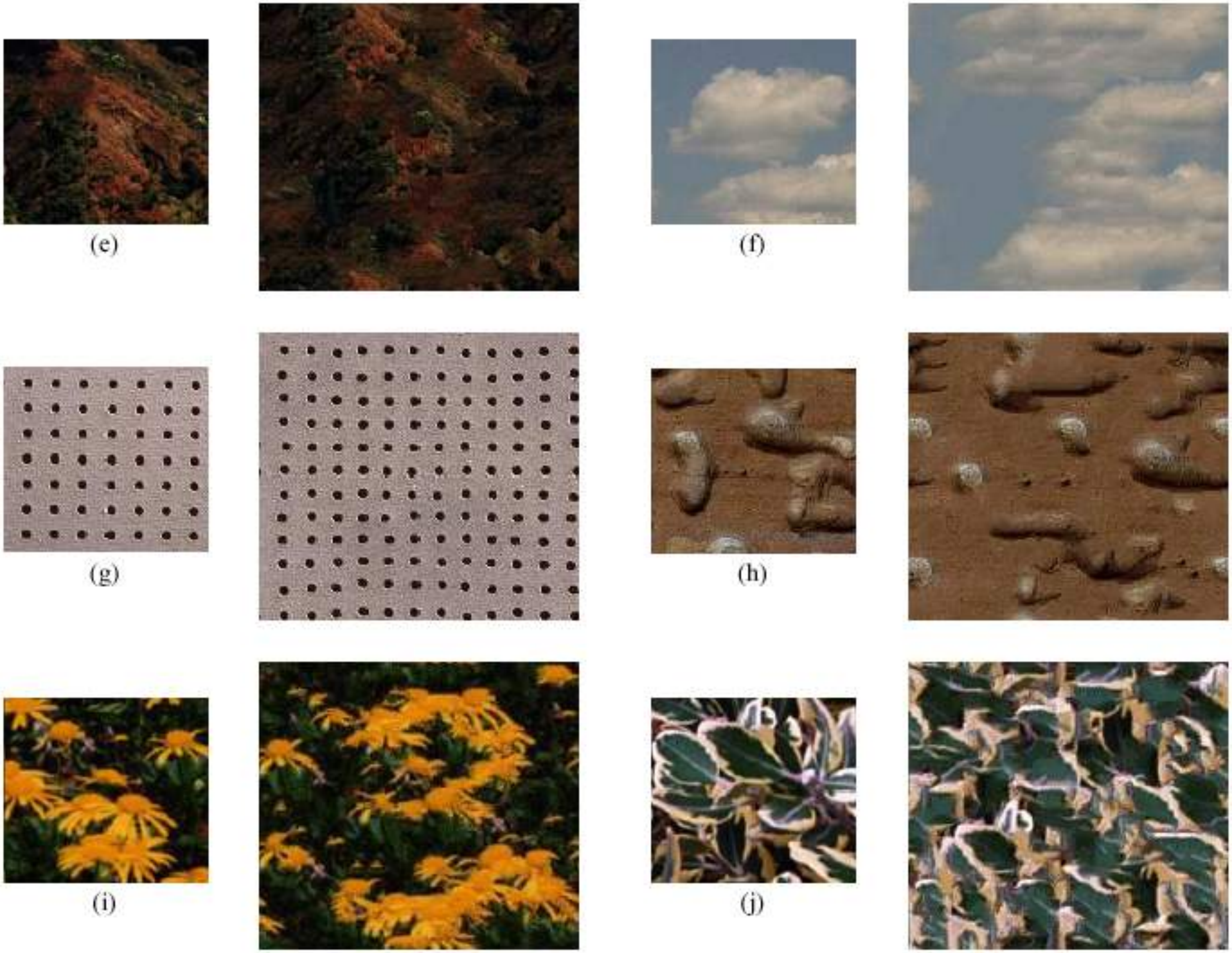
Texture sources: Photographs

Texture sources: Solid textures

Texture sources: Procedural



Texture sources: Synthesized



Kwatra et al, SIGGRAPH '05

Original

Synthesized

Original

Synthesized

Topic 1:

Texture Mapping

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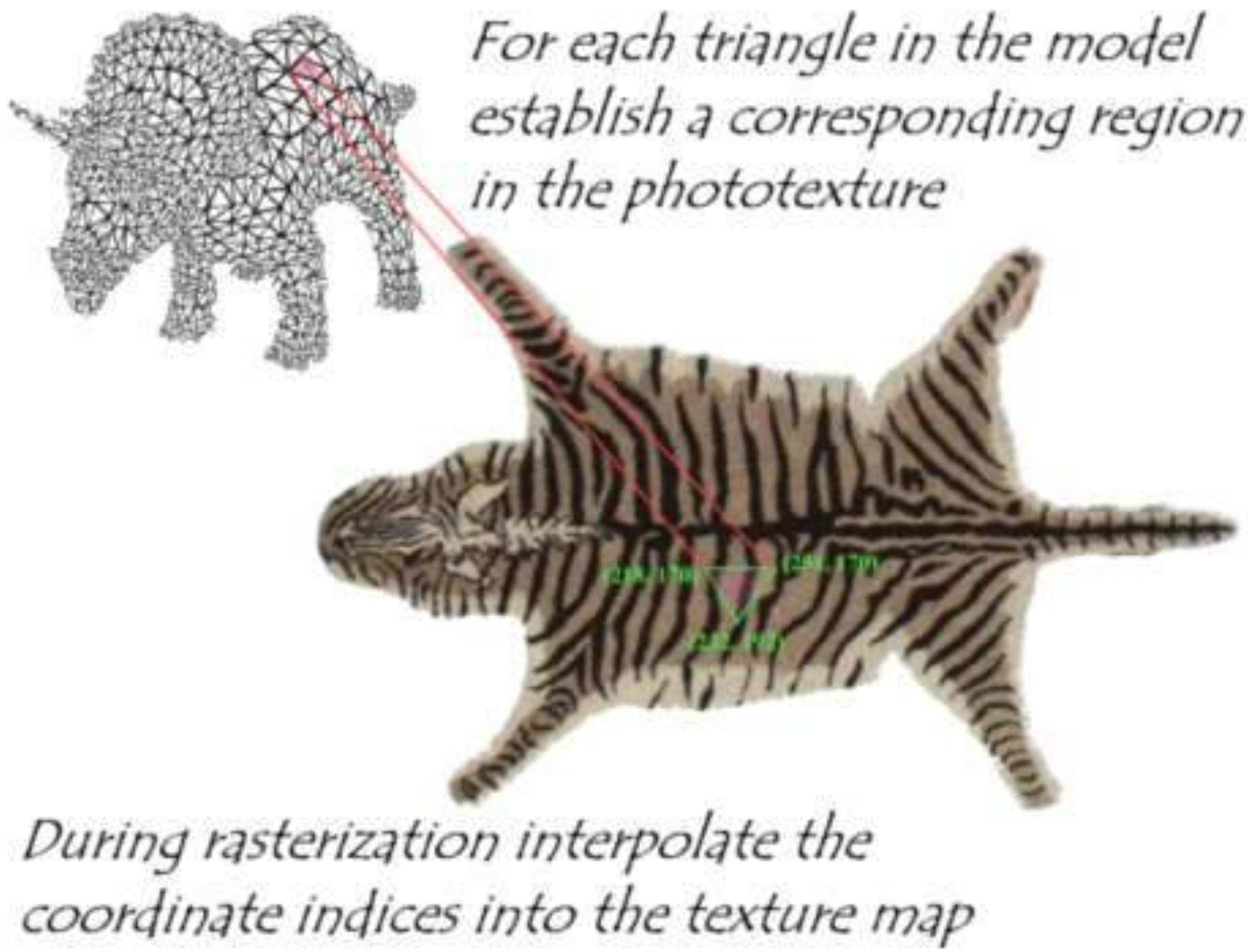
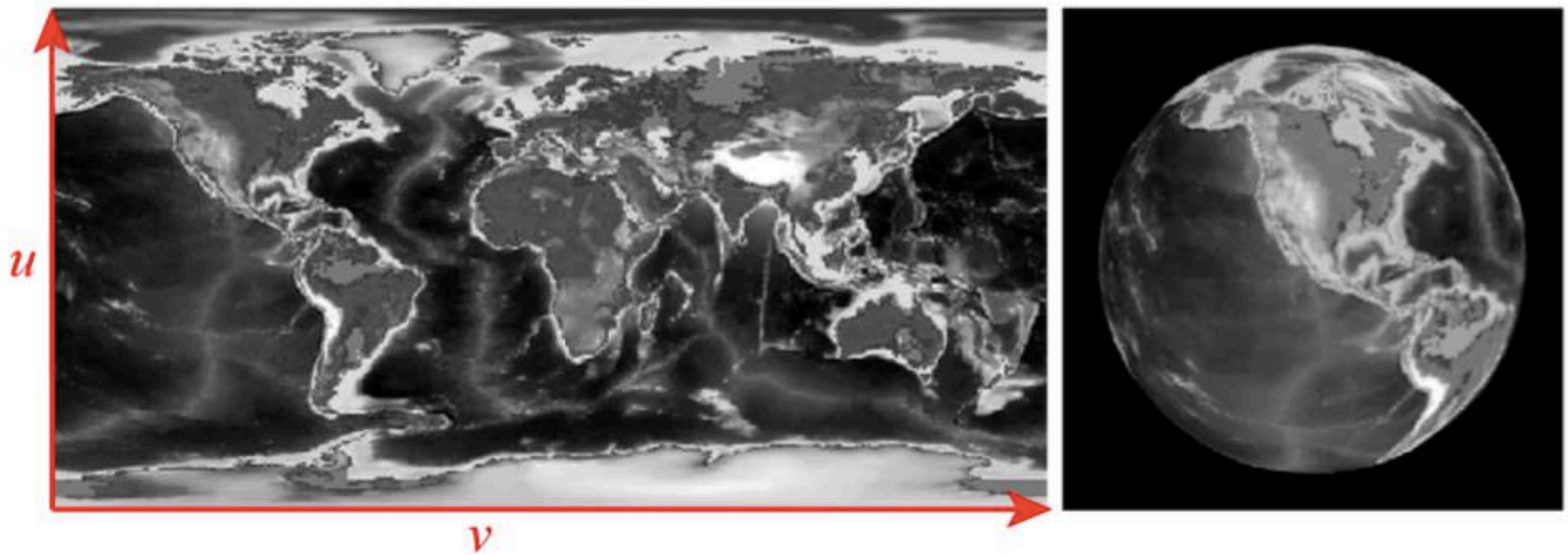
Texture coordinates

How does one establish correspondence? (UV mapping)



Texture coordinates

Example: use world map and sphere to create a globe



Per conventions we usually assume $u, v \in [0, 1]$.

Texture coordinates

$$
\begin{aligned}
x &= x_c + r\cos\phi\sin\theta \\
y &= y_c + r\sin\phi\sin\theta \\
z &= z_c + r\cos\theta
\end{aligned}
$$

Given a point (x, y, z) on the surface of the sphere, we can find θ and ϕ by

$$
\begin{aligned}
\theta &= \arccos\tfrac{z-z_c}{r} \quad \text{(cf. longitude)} \\
\phi &= \arctan\tfrac{y-y_c}{x-x_c} \quad \text{(cf. latitude)}
\end{aligned}
$$

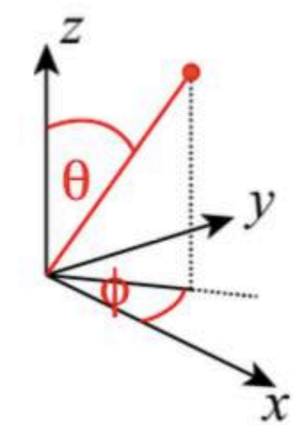
(Note: $\arccos$ is the inverse of $\cos$, $\arctan$ is the inverse of $\tan = \frac{\sin}{\cos}$)

Texture coordinates

For a point (x, y, z) we have

$$\theta = \arccos \frac{z - z_c}{r}$$
$$\phi = \arctan \frac{y - y_c}{x - x_c}$$

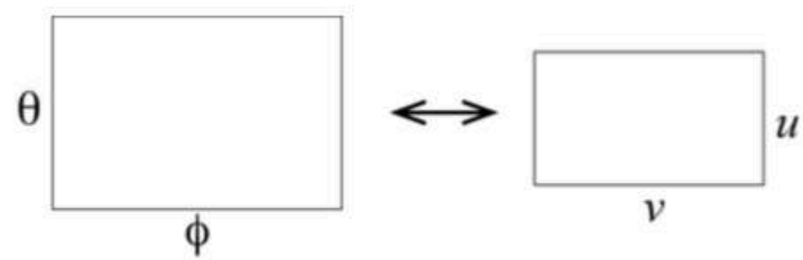
$(\theta, \phi) \in [0, \pi] \times [-\pi, \pi]$, and u, v must range from $[0, 1]$.



Hence, we get:

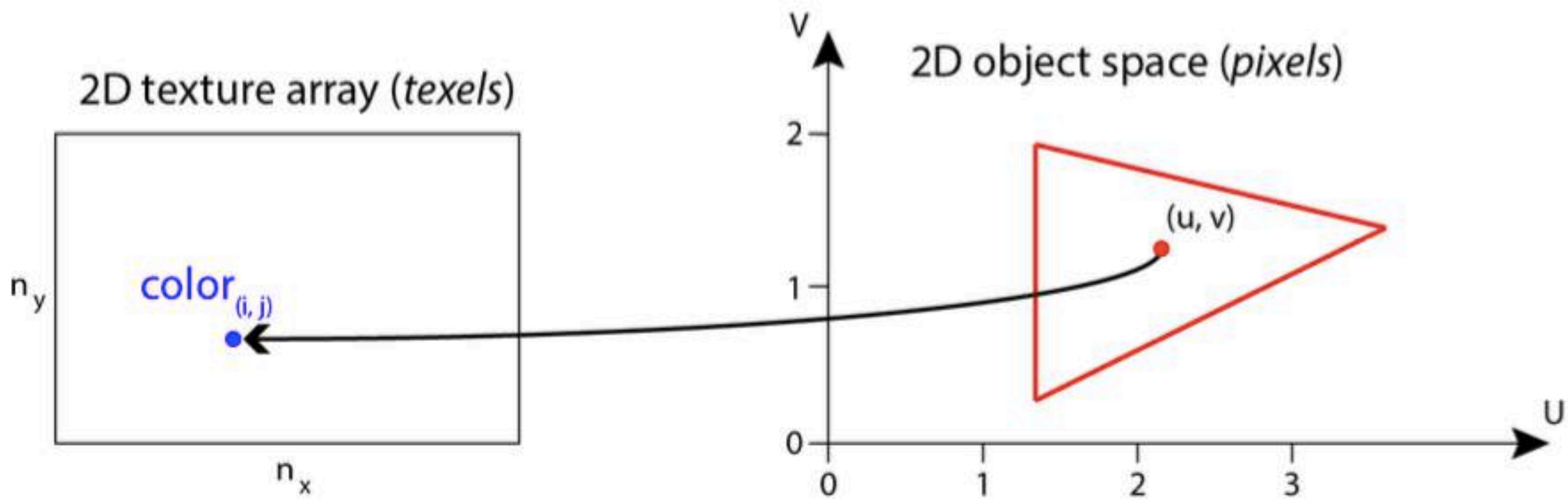
$$u = \frac{\phi \mod 2\pi}{2\pi}$$
$$v = \frac{\pi - \theta}{\pi}$$

(Note that this is a simple scaling transformation in 2D)



Texture coordinates

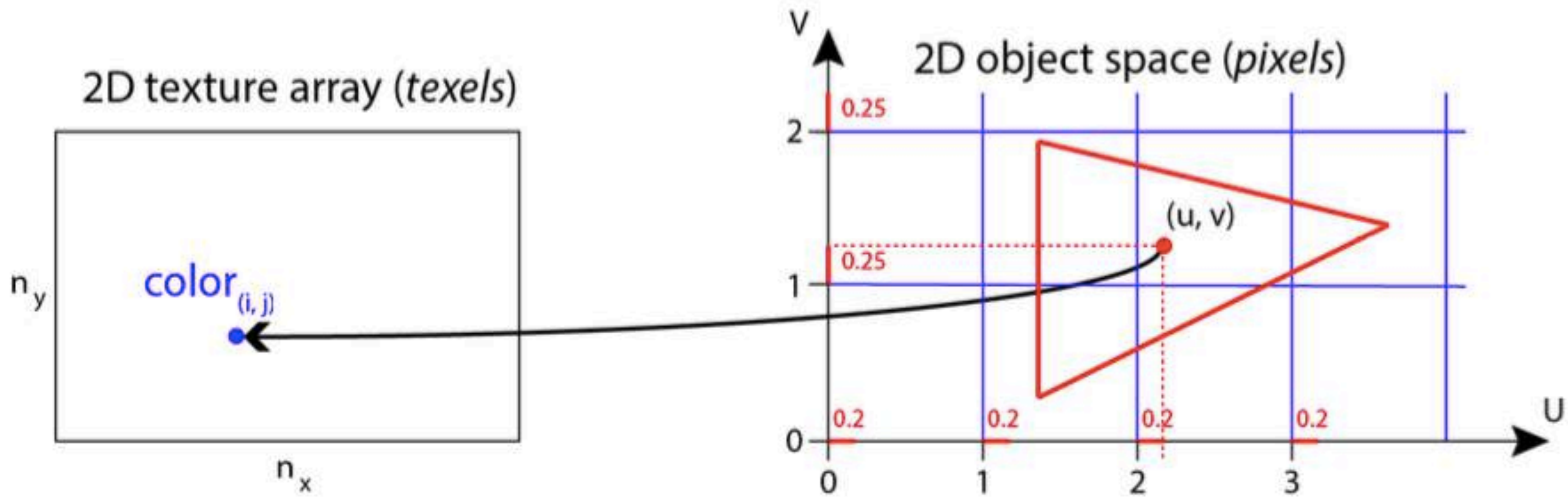
Example: "Tiling" of 2D textures into a UV-object space



We'll call the two dimensions to be mapped u and v, and assume an $n_x \times n_y$ image as texture.

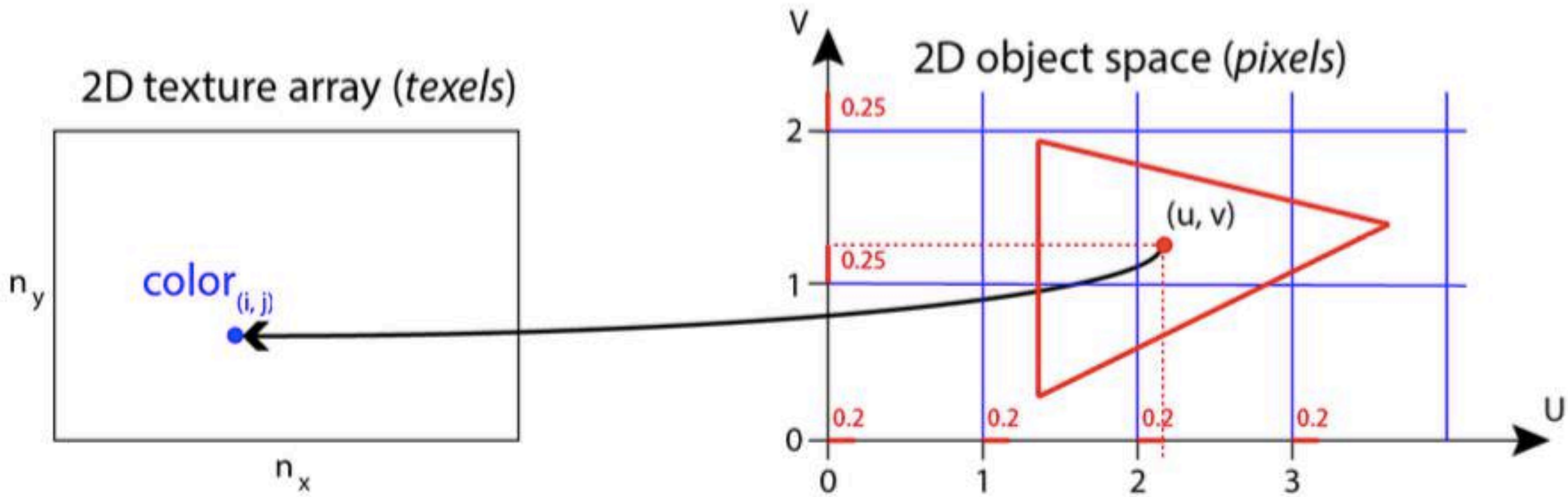
Then every (u, v) needs to be mapped to a color in the image, i.e. we need a mapping from pixels to texels.

Texture coordinates



A standard way is to first
remove the integer portion of u and v,
so that (u, v) lies in the unit square.

Texture coordinates

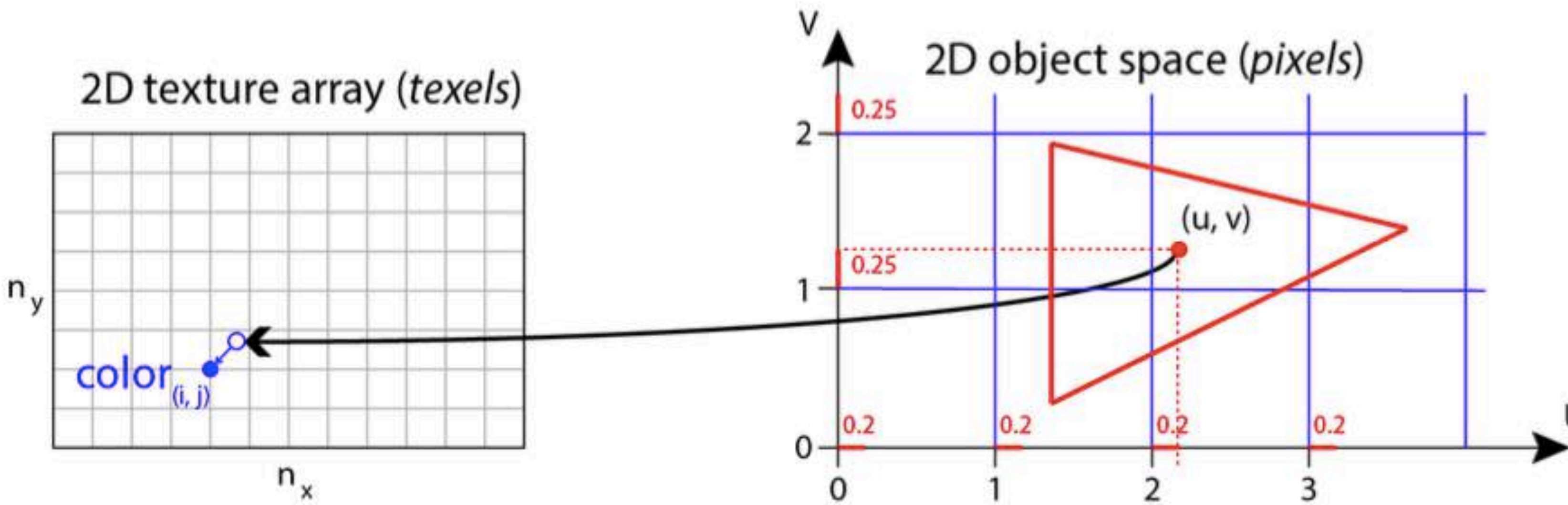


This results in a simple mapping from $0 \leq u, v \leq 1$ to the size of the texture array, i.e. $n_x \times n_y$.

$$i = un_x \text{ and } j = vn_y$$

Yet, for the array lookup, we need integer values.

Texture coordinates



The texel (i, j) in the $n_x \times n_y$ image for (u, v) can be determined using the floor function $\lfloor x \rfloor$ which returns the highest integer value $\leq x$.

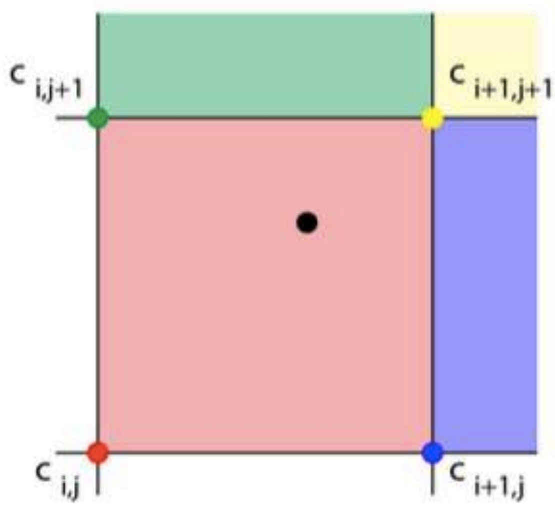
$$i = \lfloor u n_x \rfloor \text{ and } j = \lfloor v n_y \rfloor$$

Texture coordinates

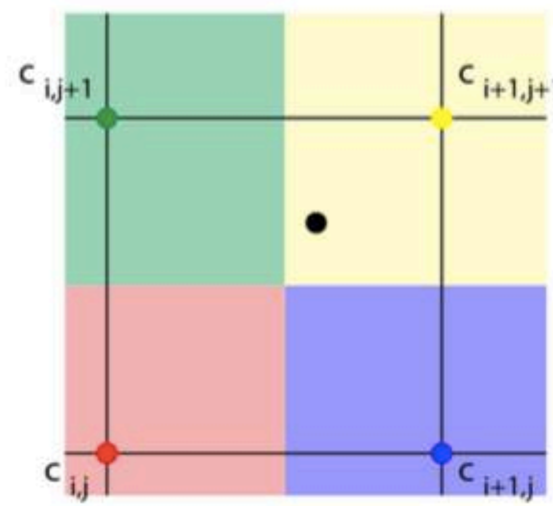
$$c(u, v) = c_{i,j} \text{ with } i = \lfloor un_x \rfloor \text{ and } j = \lfloor vn_y \rfloor$$

This is a version of nearest-neighbor interpolation, where we take the color of the nearest neighbor.

Floor function



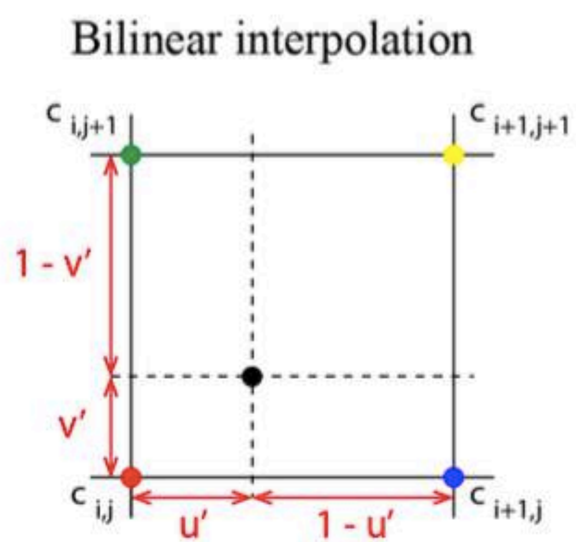
Nearest neighbor mapping



Texture coordinates

For smoother effects we may use bilinear interpolation:

$$c(u,v) =$$
$$(1-u')(1-v')c_{ij}+u'(1-v')c_{(i+1)j}+(1-u')v'c_{i(j+1)}+u'v'c_{(i+1)(j+1)}$$



with
$u' = un_x - \lfloor un_x \rfloor$ and
$v' = vn_y - \lfloor vn_y \rfloor$

Notice that all weights are between 0 and 1 and add up to 1:
$$(1-u')(1-v') + u'(1-v')+$$
$$(1-u')v' + u'v' = 1$$

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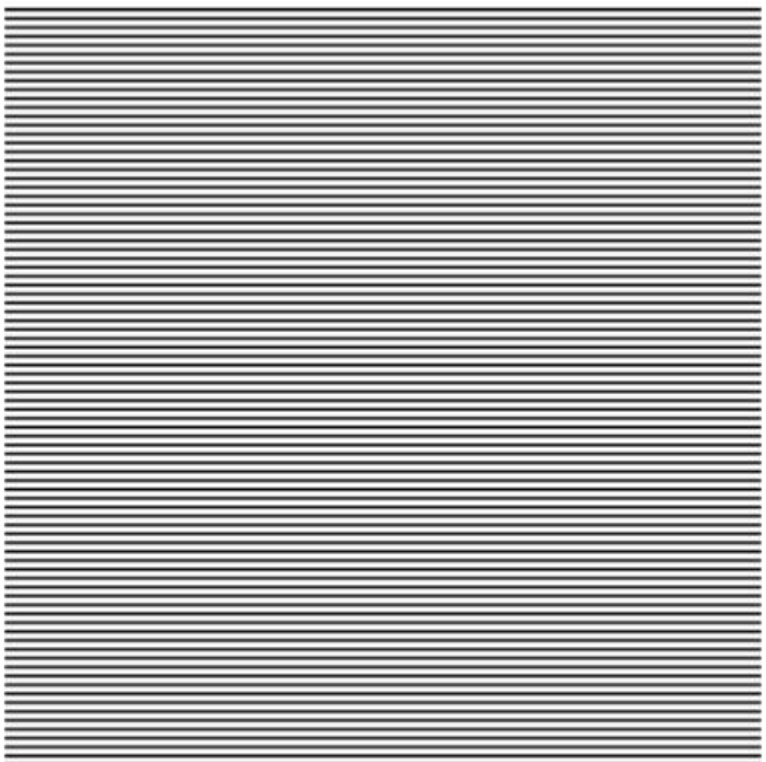
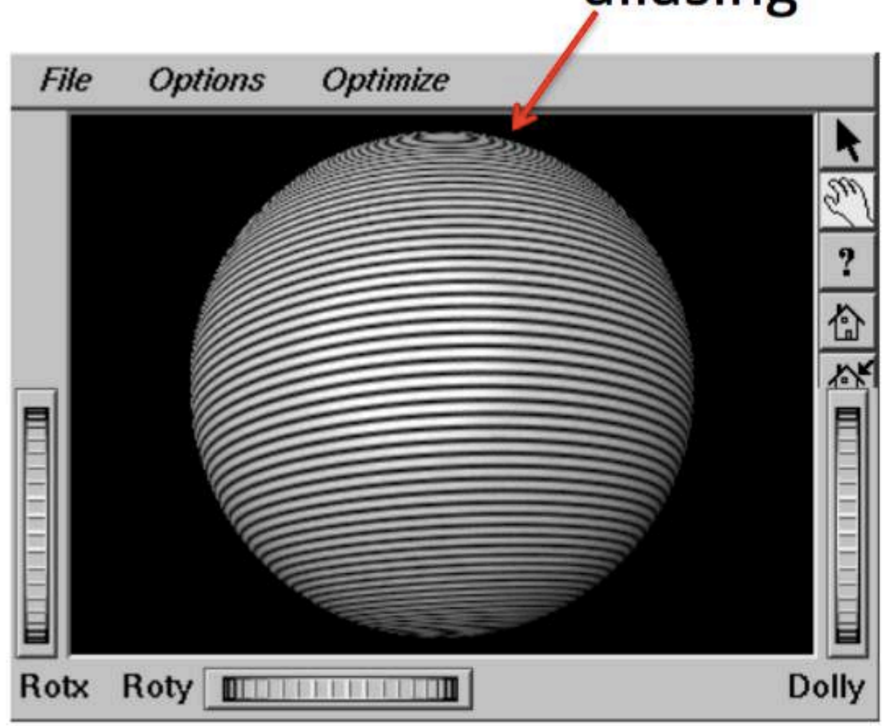
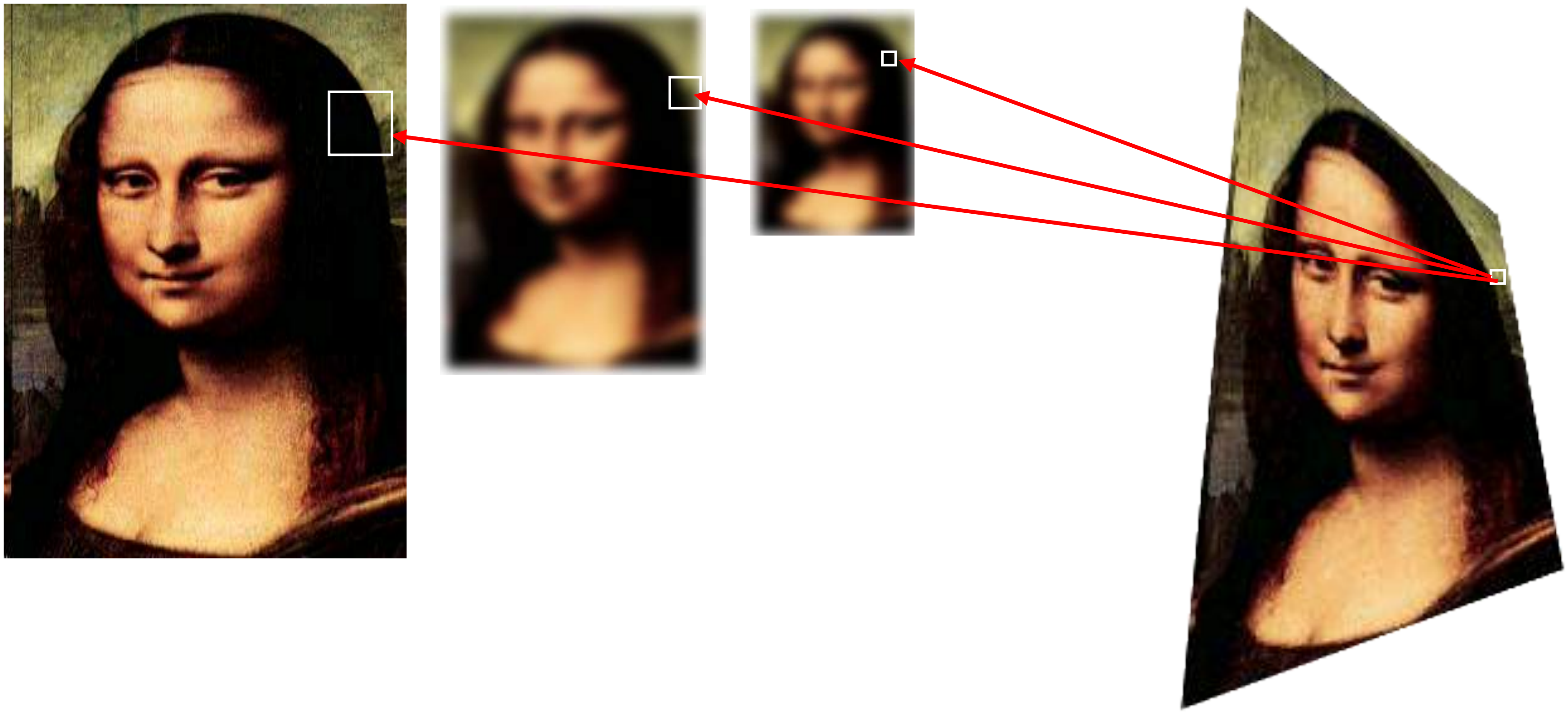
Mipmapping



aliasing

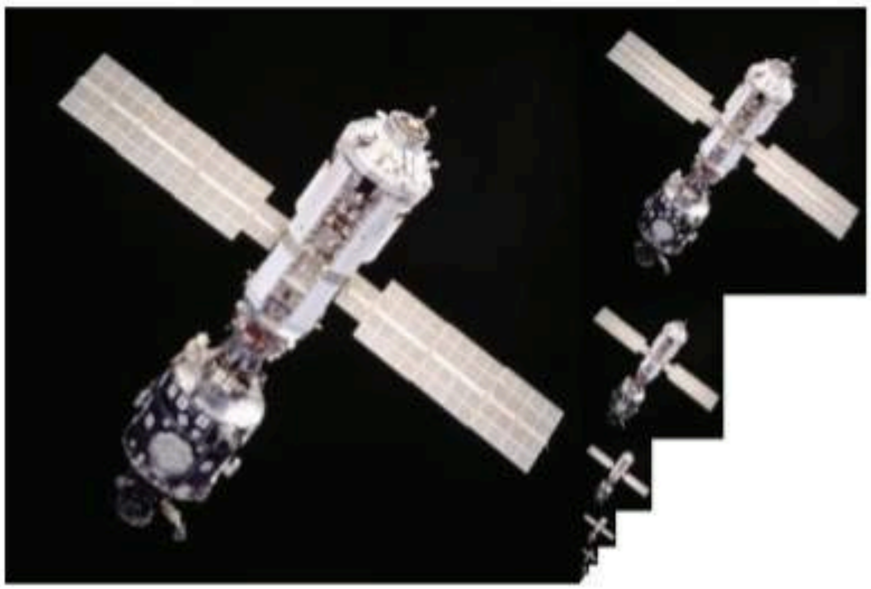


MIP-Mapping: Basic Idea



Given a polygon, use the texture image, where the projected polygon best matches the size of the polygon on screen.

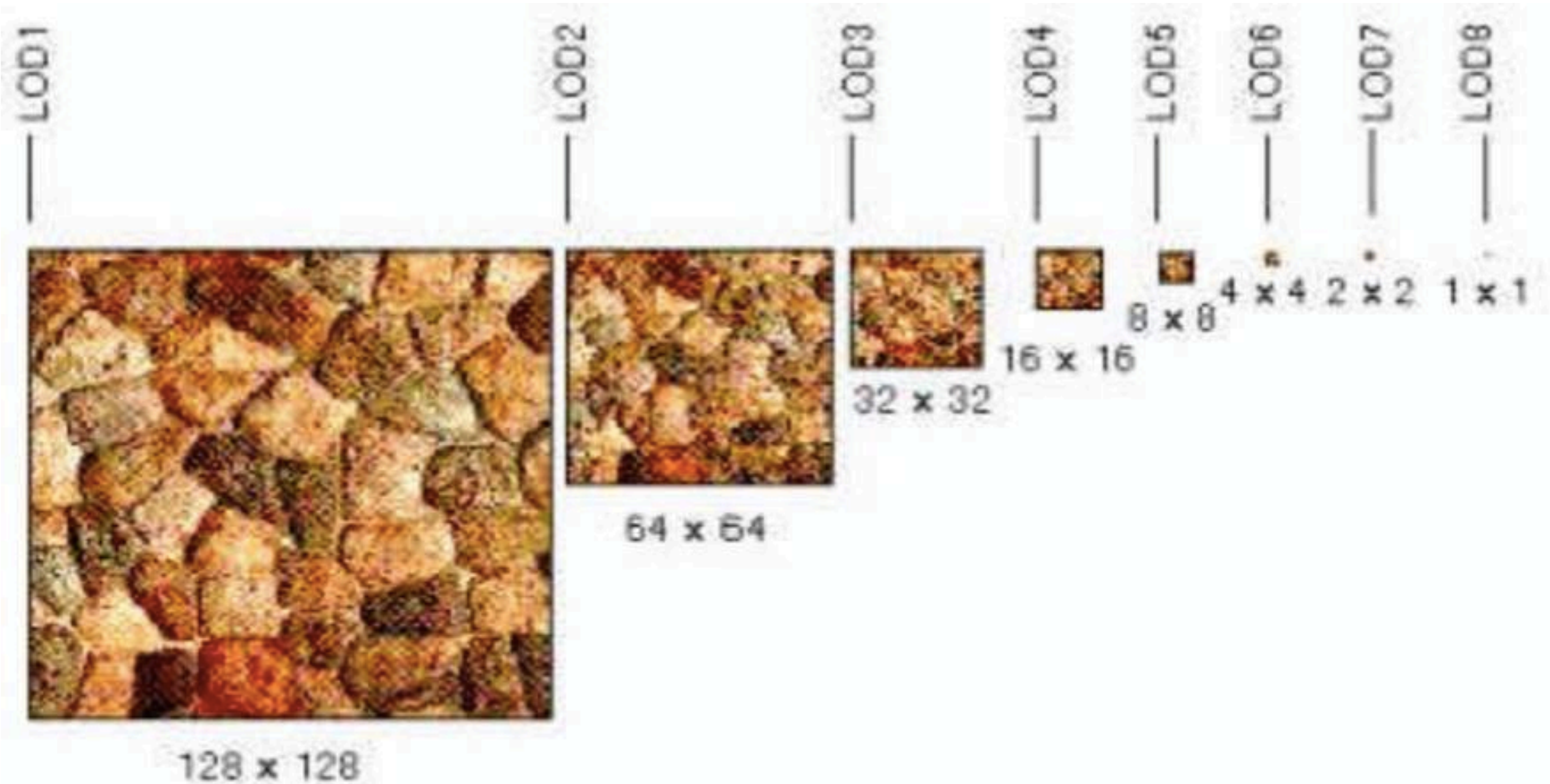
Mipmapping



Solutions: MIP maps

- Pre-calculated, optimized collections of images based on the original texture
- Dynamically chosen based on depth of object (relative to viewer)
- Supported by todays hardware and APIs

Mipmapping


LOD1
LOD2
LOD3
LOD4
LOD5
LOD6
LOD7
LOD8
128 x 128
64 x 64
32 x 32
16 x 16
8 x 8
4 x 4
2 x 2
1 x 1

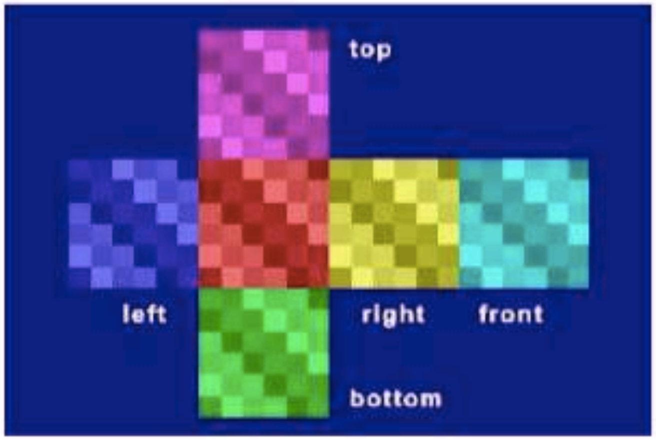
Environment mapping

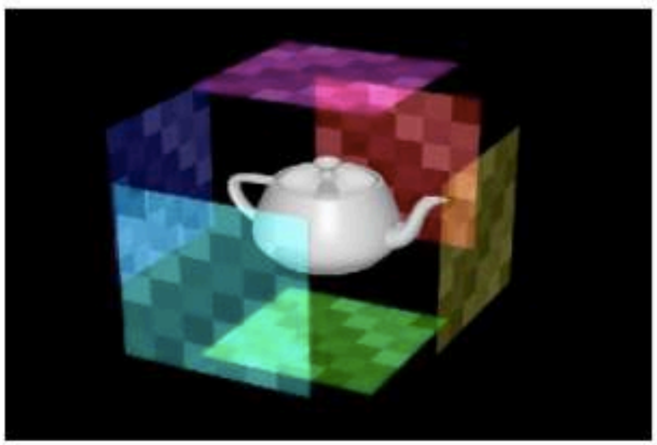
... why not use this to make objects appear to reflect their surroundings specularly?

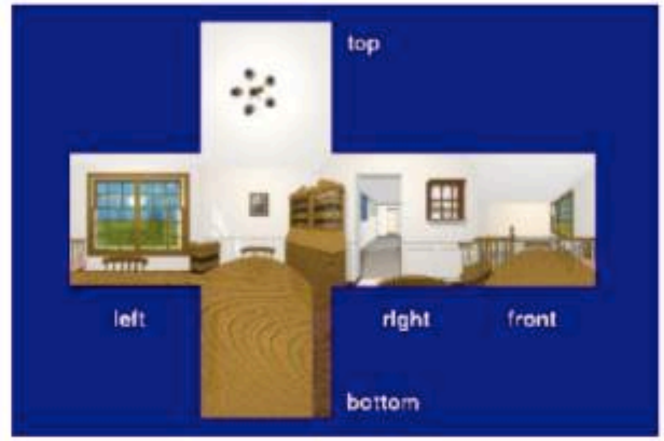
Idea: place a cube around the object, and project the environment of the object onto the planes of the cube in a preprocessing stage; this is our texture map.

During rendering, we compute a reflection vector, and use that to look-up texture values from the cubic texture map.

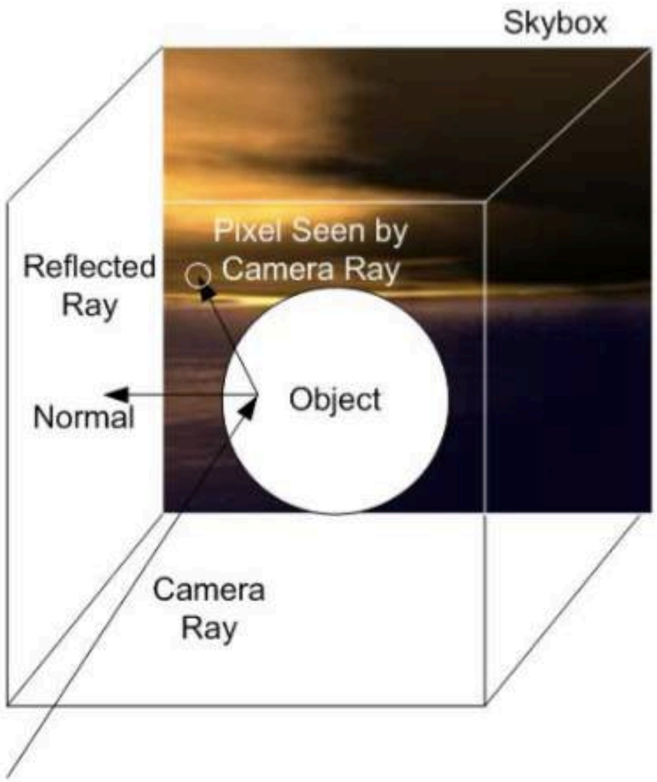
Environment mapping







Environment mapping



Remember Phong shading: "perfect" reflection if

angle between eye vector $\vec{e}$ and $\vec{n}$ = angle between $\vec{n}$ and reflection vector $\vec{r}$

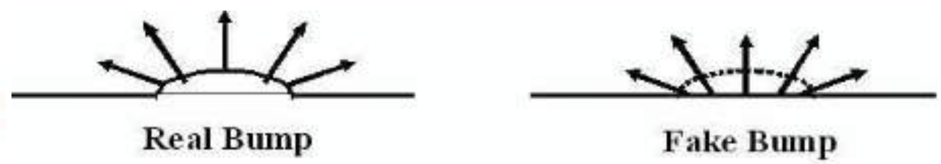
Environment mapping

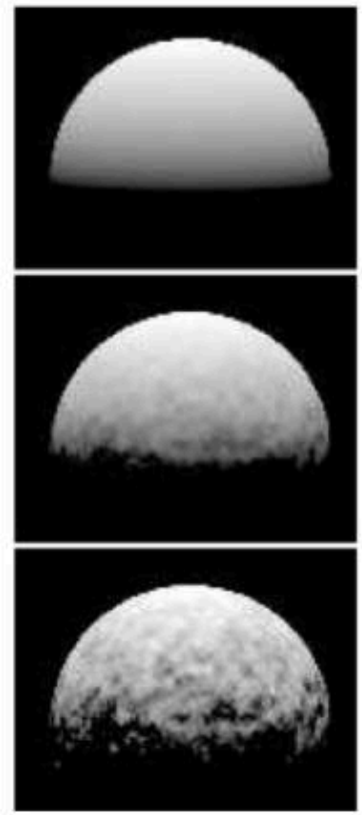
Image from slides by

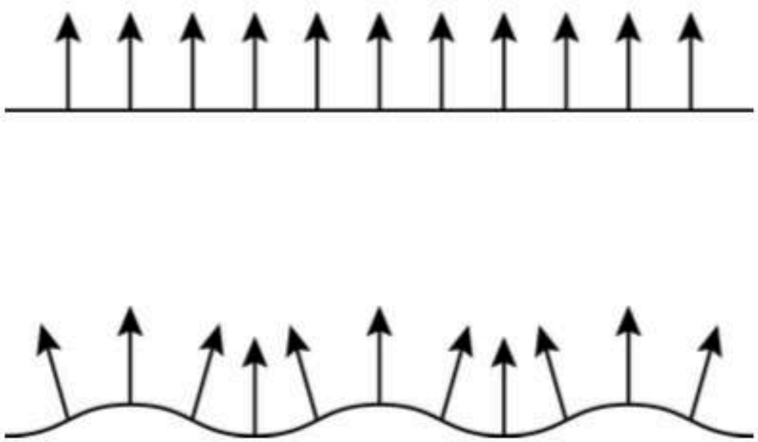
Bump mapping

One of the reasons why we apply texture mapping:

Real surfaces are hardly flat but often rough and bumpy. These bumps cause (slightly) different reflections of the light.

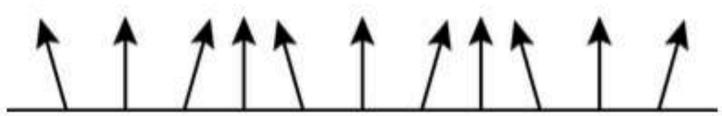


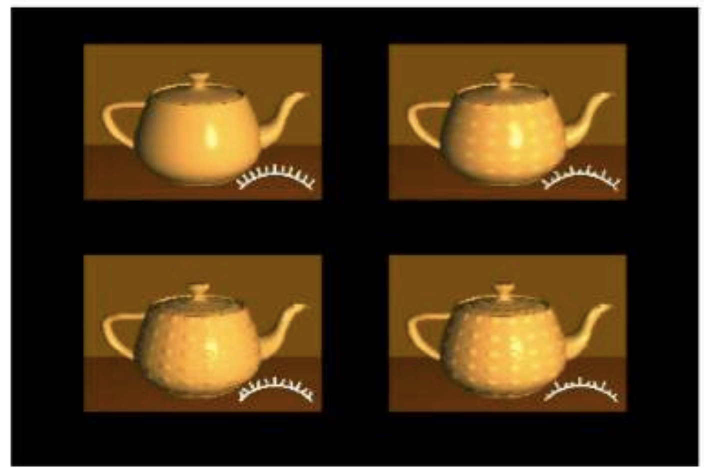




Bump mapping

Instead of mapping an image or noise onto an object, we can also apply a bump map, which is a 2D or 3D array of vectors. These vectors are added to the normals at the points for which we do shading calculations.

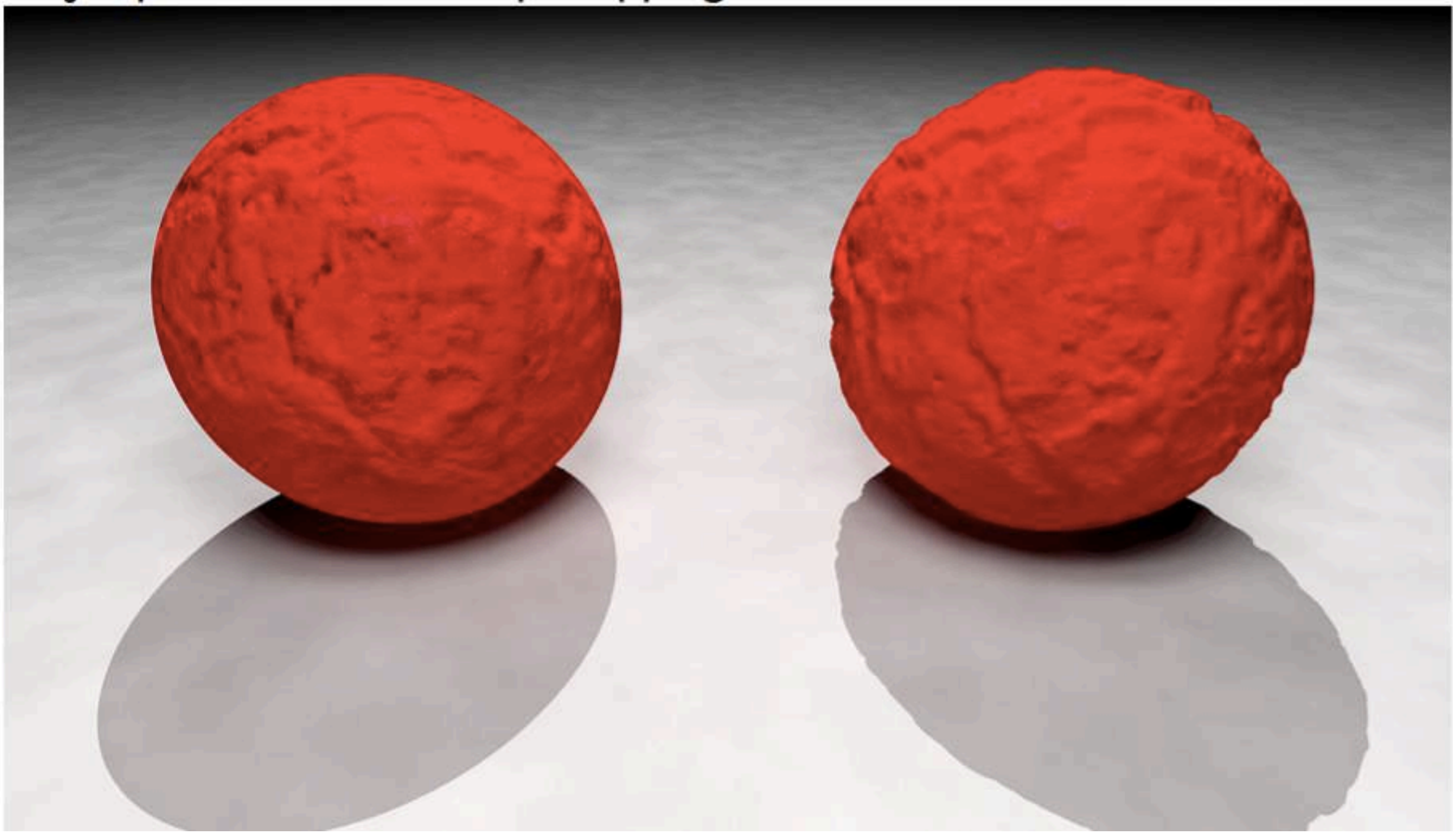




The effect of bump mapping is an apparent change of the geometry of the object.

Bump mapping

Major problems with bump mapping: silhouettes and shadows

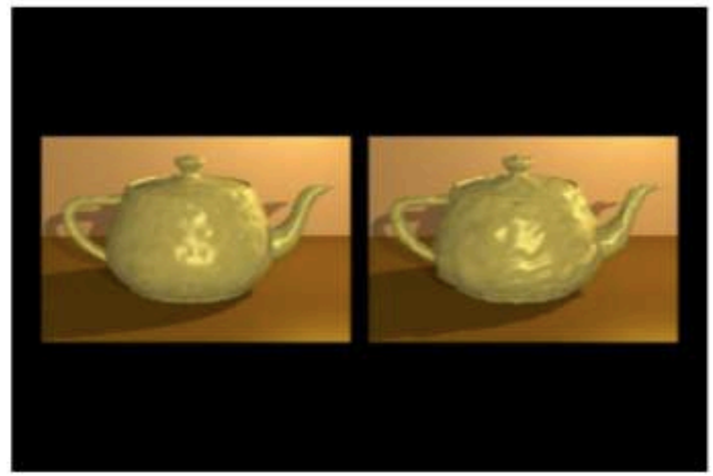



2D Image Bump Mapping Using a 24-bit Bitmap

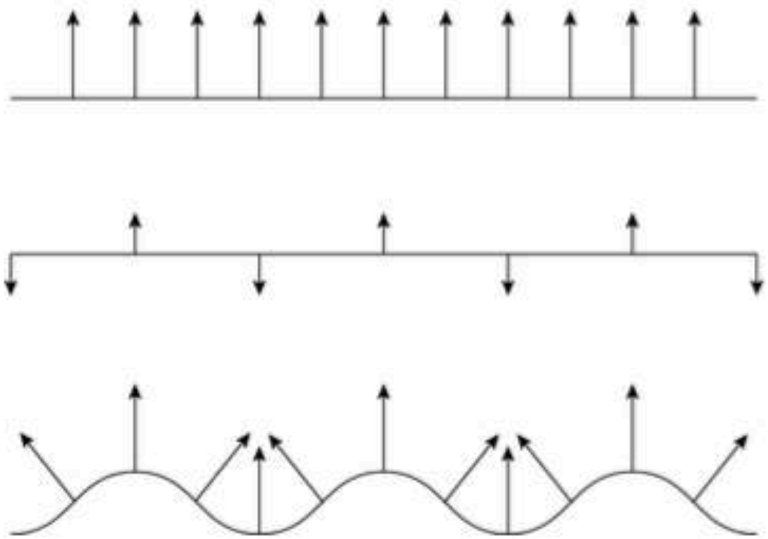
Displacement mapping

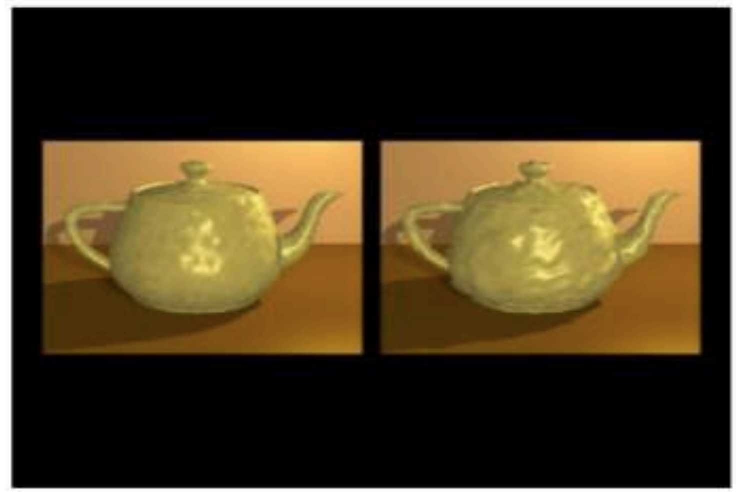
To overcome this shortcoming, we can use a displacement map. This is also a 2D or 3D array of vectors, but here the points to be shaded are actually displaced.

Normally, the objects are refined using the displacement map, giving an increase in storage requirements.

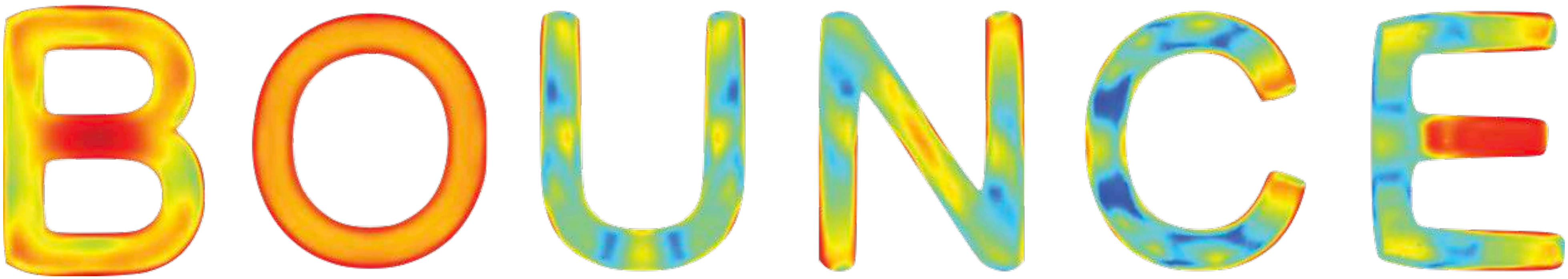


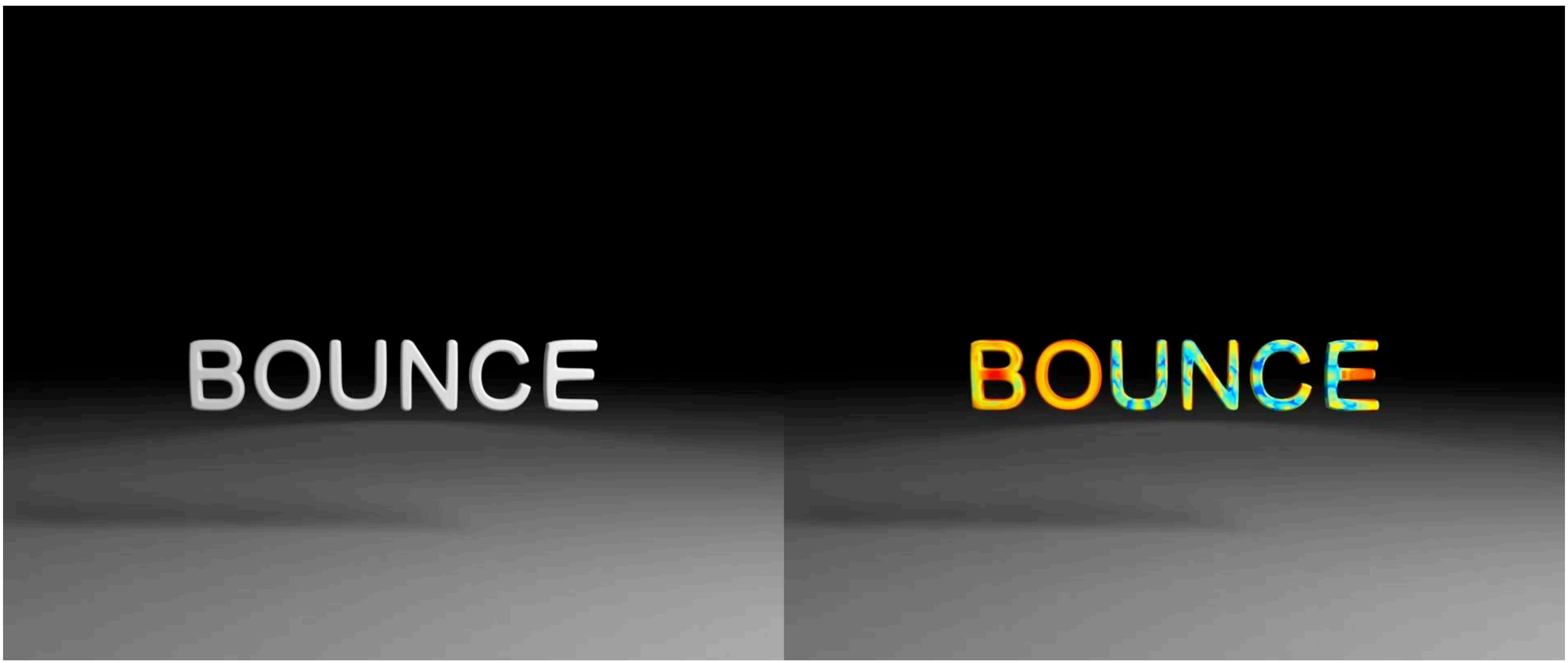
Displacement mapping





Bounce Maps





Topic 2:

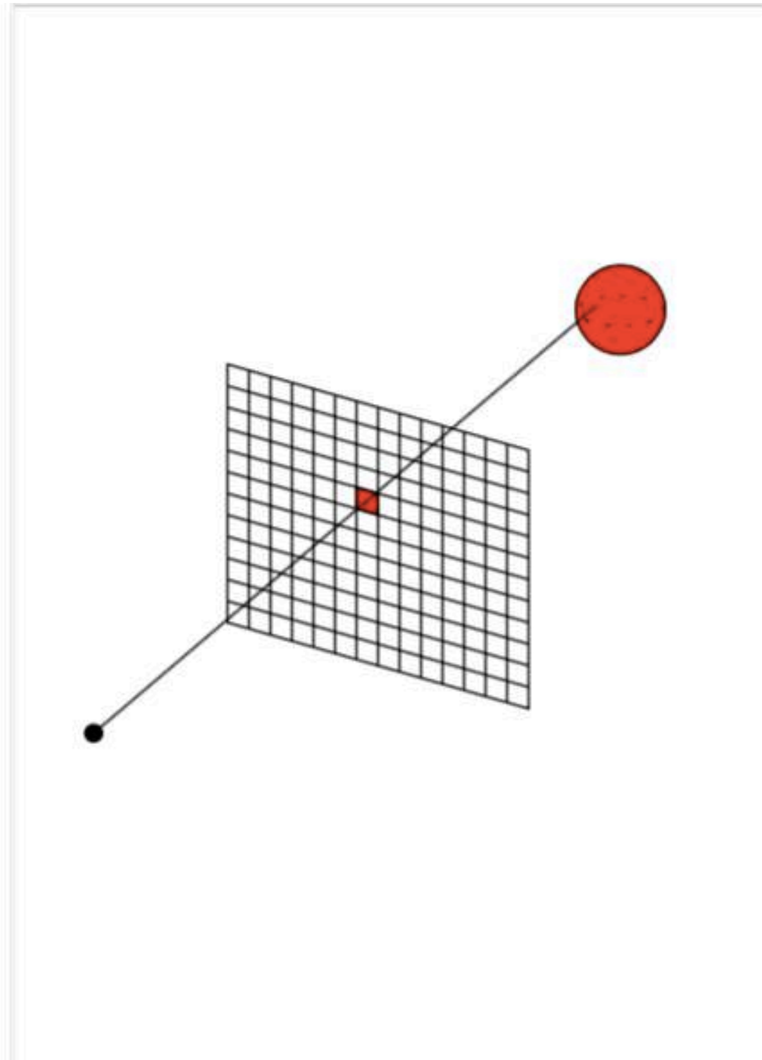
Basic Ray Tracing

- Introduction to ray tracing
- Computing rays
- Computing intersections
  - ray-triangle
  - ray-polygon
  - ray-quadric
  - the scene signature
- Computing normals
- Evaluating shading model
- Spawning rays
- Incorporating transmission
  - refraction
  - ray-spawning & refraction

A basic ray tracing algorithm

FOR each pixel DO

- compute viewing ray
- find the 1st object hit
  by the ray
  and its surface normal $\vec{n}$
- set pixel color to value
  computed from hit point,
  light, and $\vec{n}$



Shading model

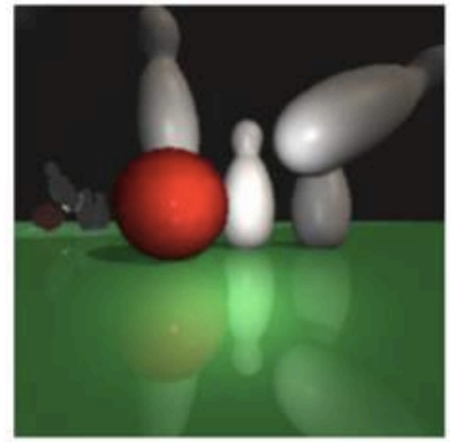
Remember our shading model:

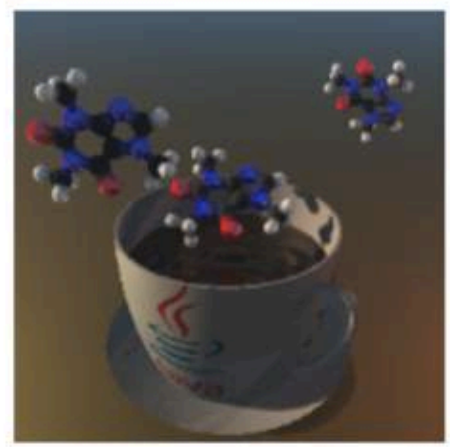
$$c = c_r(c_a + c_l \max(0, n \cdot l)) + c_l(\vec{h} \cdot \vec{n})^p$$

with

- Ambient shading
- Lambertian shading
- Phong shading

and Gouraud interpolation.

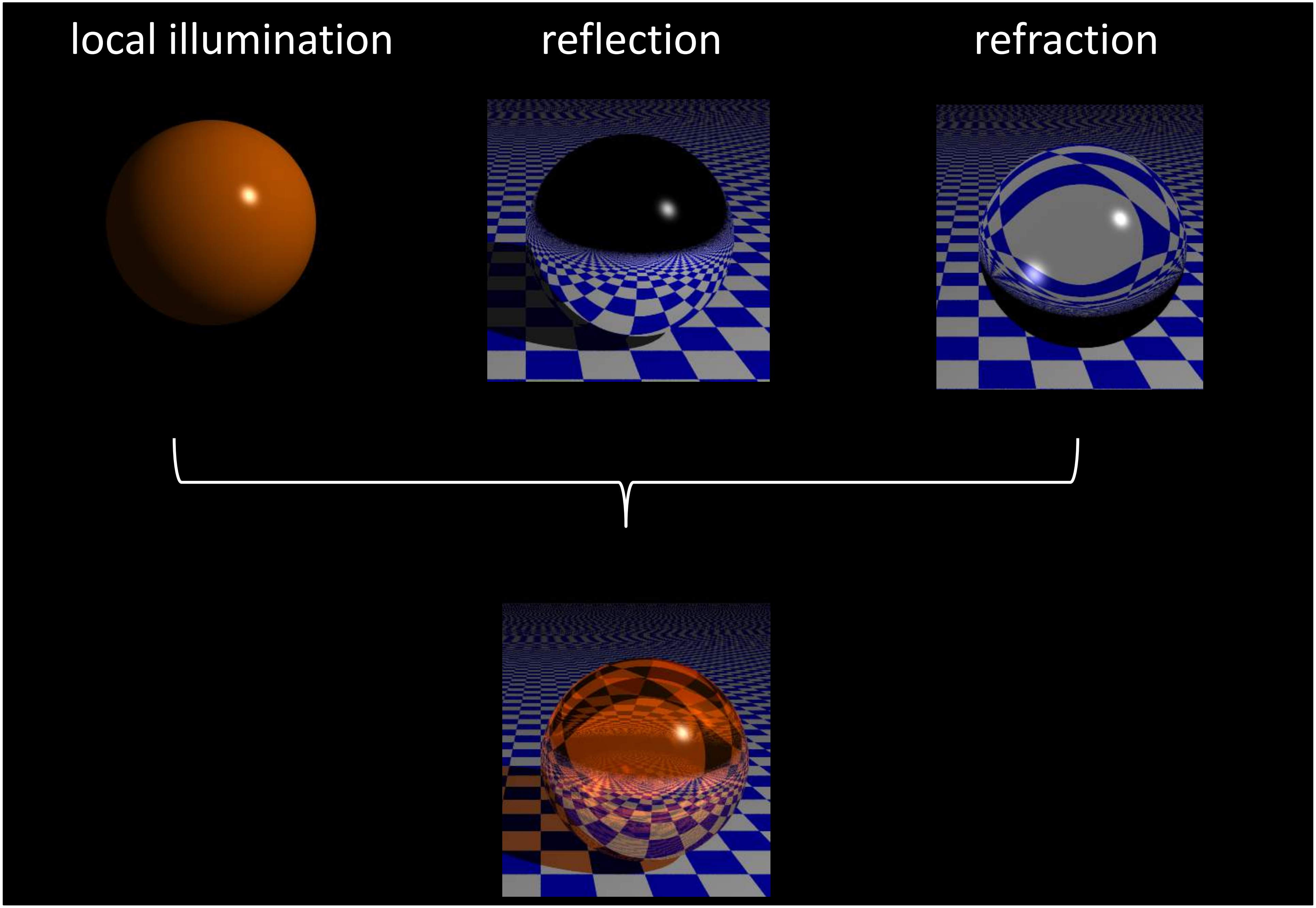




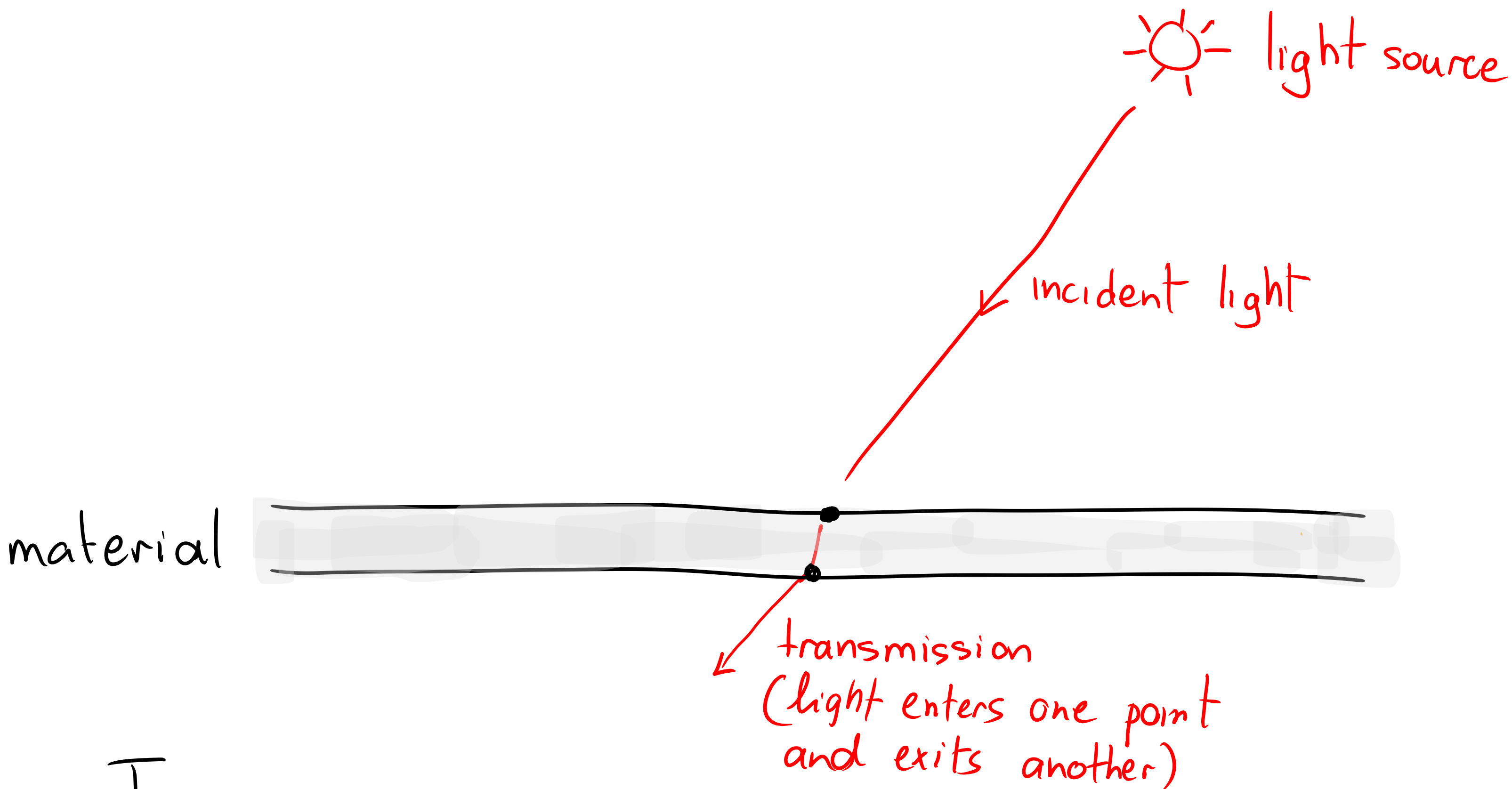
Topic 3:

Less Basic Ray Tracing

- Introduction to ray tracing
- Computing rays
- Computing intersections
  - ray-triangle
  - ray-polygon
  - ray-quadric
  - the scene signature
- Computing normals
- Evaluating shading model
- Spawning rays
- Incorporating transmission
  - refraction
  - ray-spawning & refraction


local illumination
reflection
refraction

Modeling Reflection: Transmission

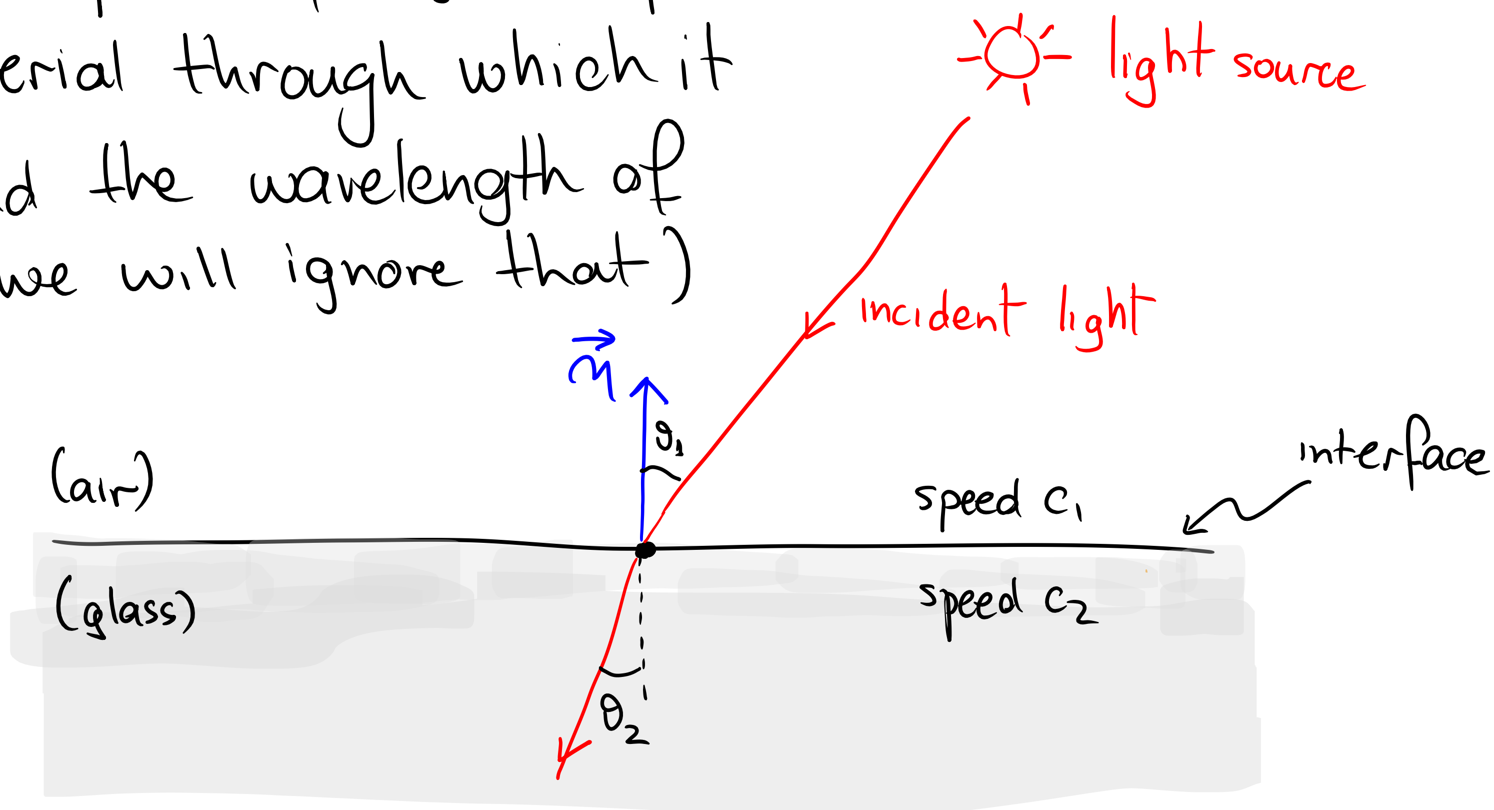


Transmission:

- Caused by materials that are not perfectly opaque
- Examples include glass, water and transluscent materials such as skin

Physics of Refraction

Physics: the speed of light depends on the material through which it travels (and the wavelength of light, but we will ignore that)

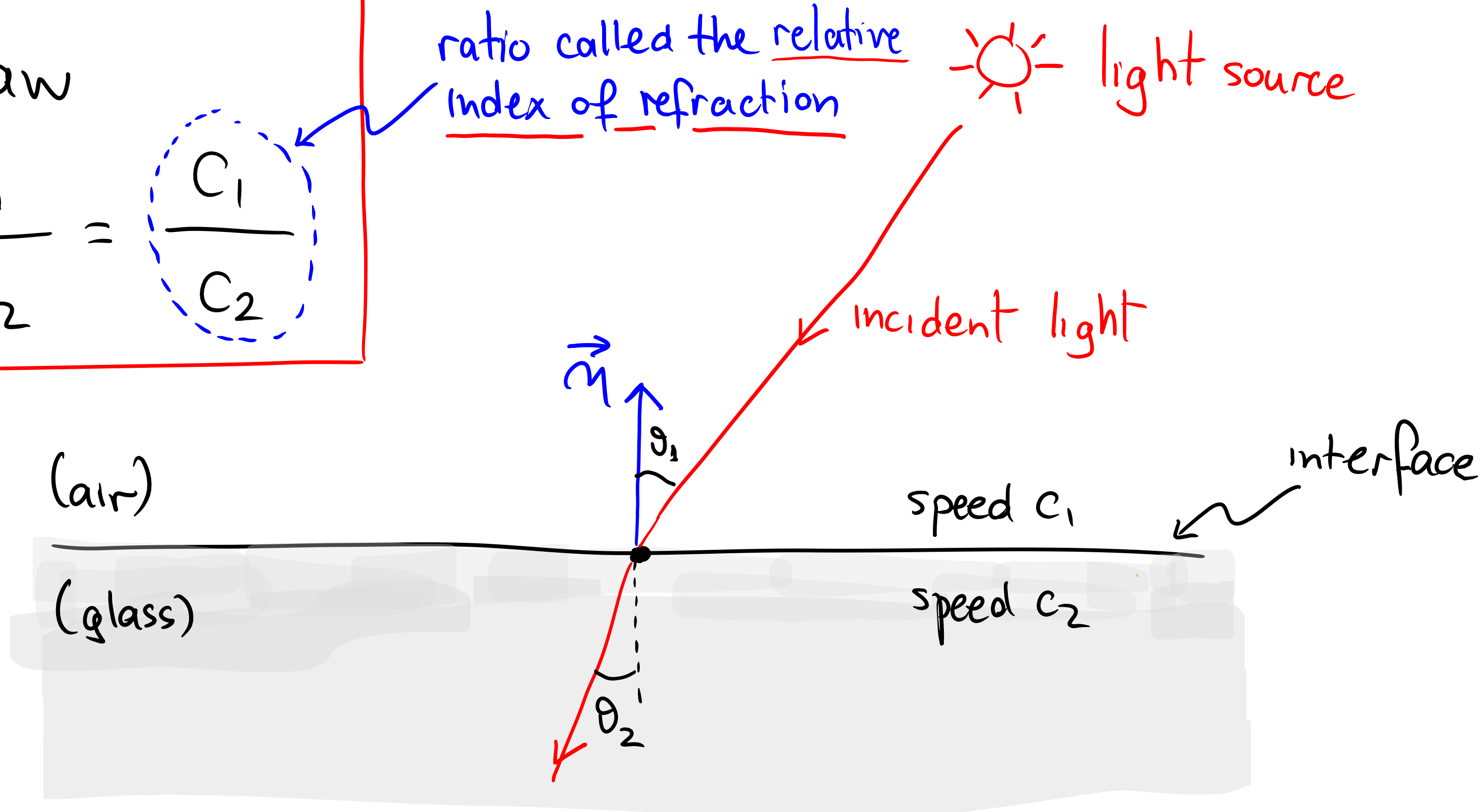


Refraction (bending of rays) occurs when light crosses an interface between two media with different speeds of light

Physics of Refraction

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$

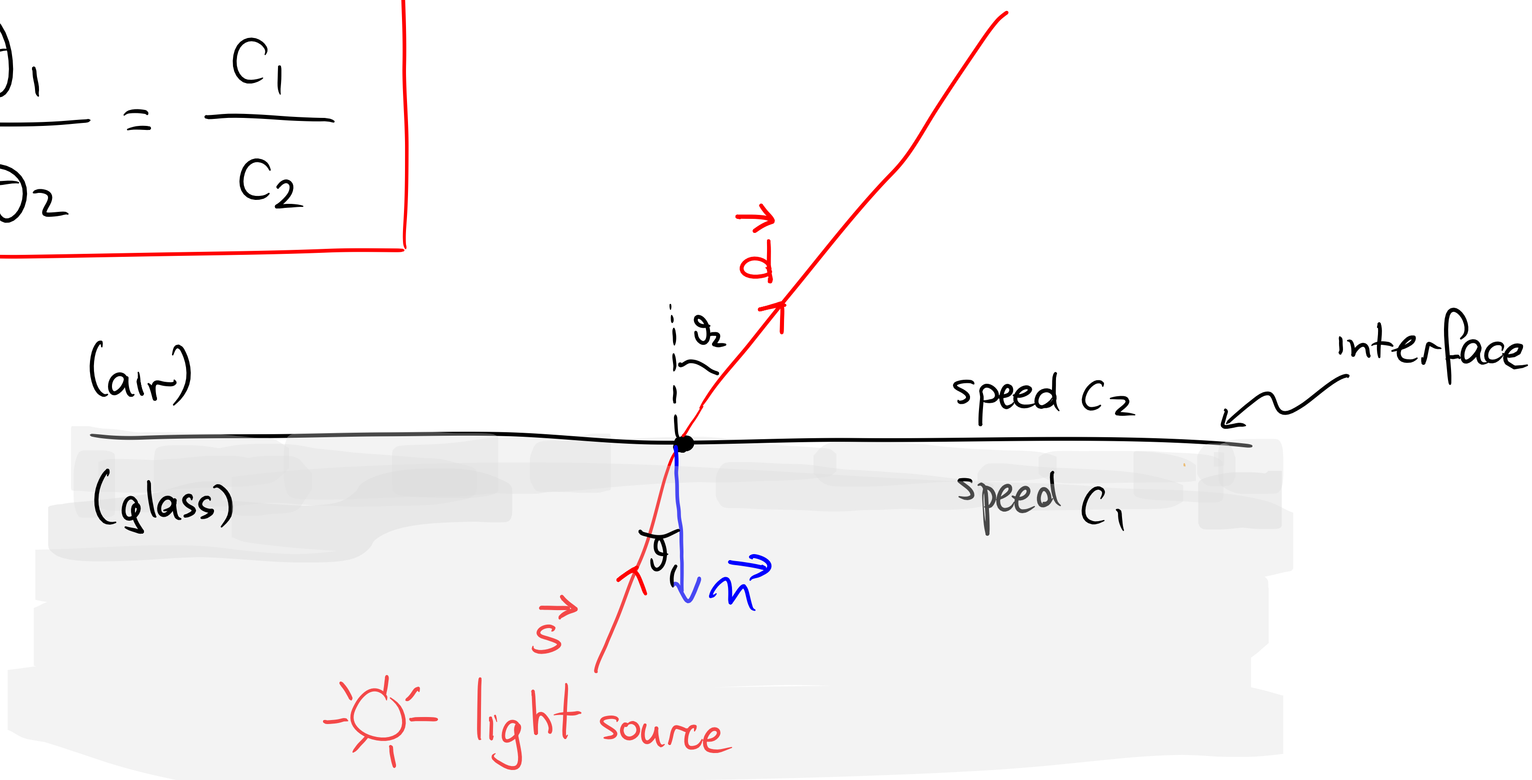


Refraction (bending of rays) occurs when light crosses an interface between two media with different speeds of light

Geometry of Refraction

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$



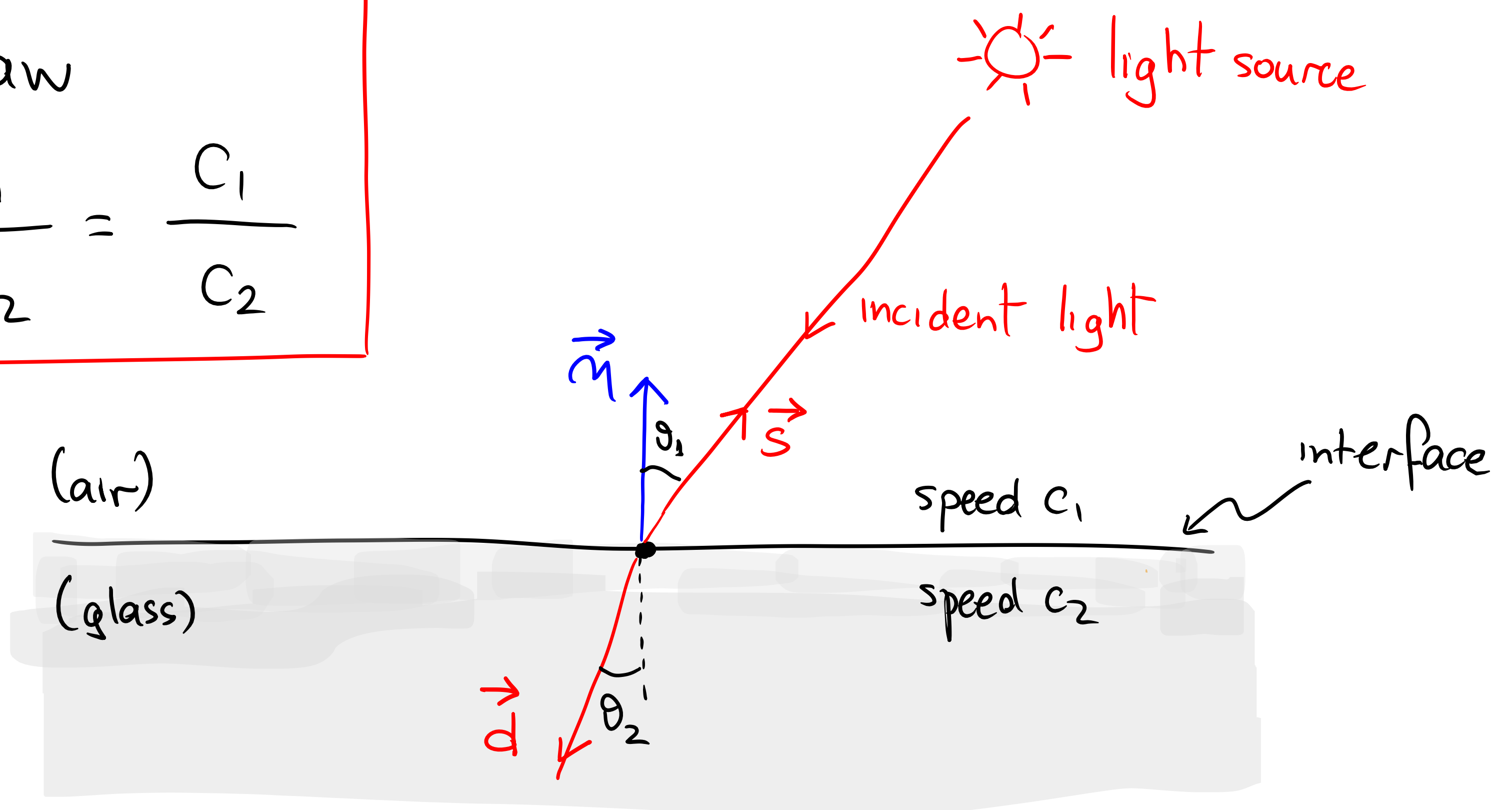
③ If $c_2 < c_1$ light bends toward the normal

If $c_2 > c_1$ light bends away from normal

Geometry of Refraction: Transmission Vector

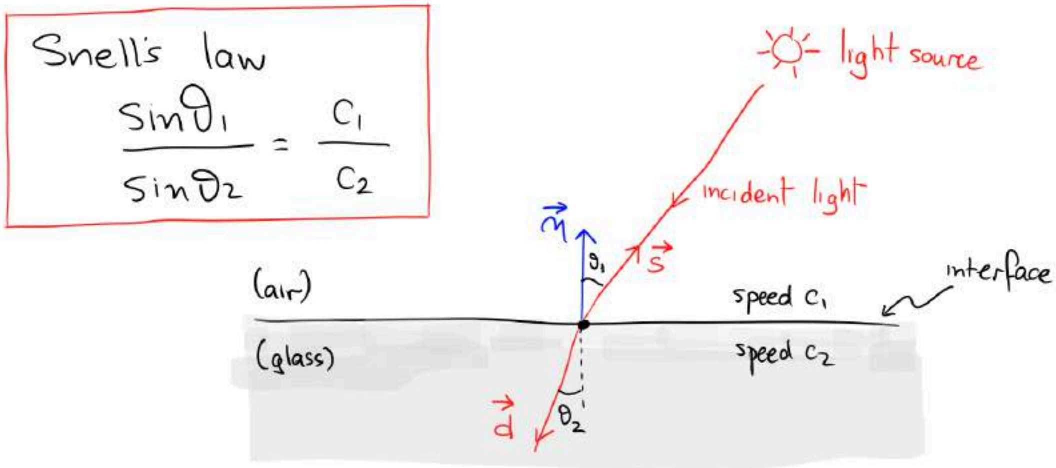
Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$



① Incident ray, outgoing ray & normal always lie on the same plane $\Rightarrow$

$$\vec{d} \text{ along } -\frac{c_2}{c_1}\vec{s} + \left[\frac{c_2}{c_1}\cos\theta_1 - \cos\theta_2\right]\vec{n}$$



Assumption: Refracted ray lies in the **same plane** as the incident ray

$$\vec{s} = \vec{s}_{\perp} + \vec{s}_{||}$$

$\vec{s}_{\perp}$ Perpendicular component

$\vec{s}_{||}$ Parallel component

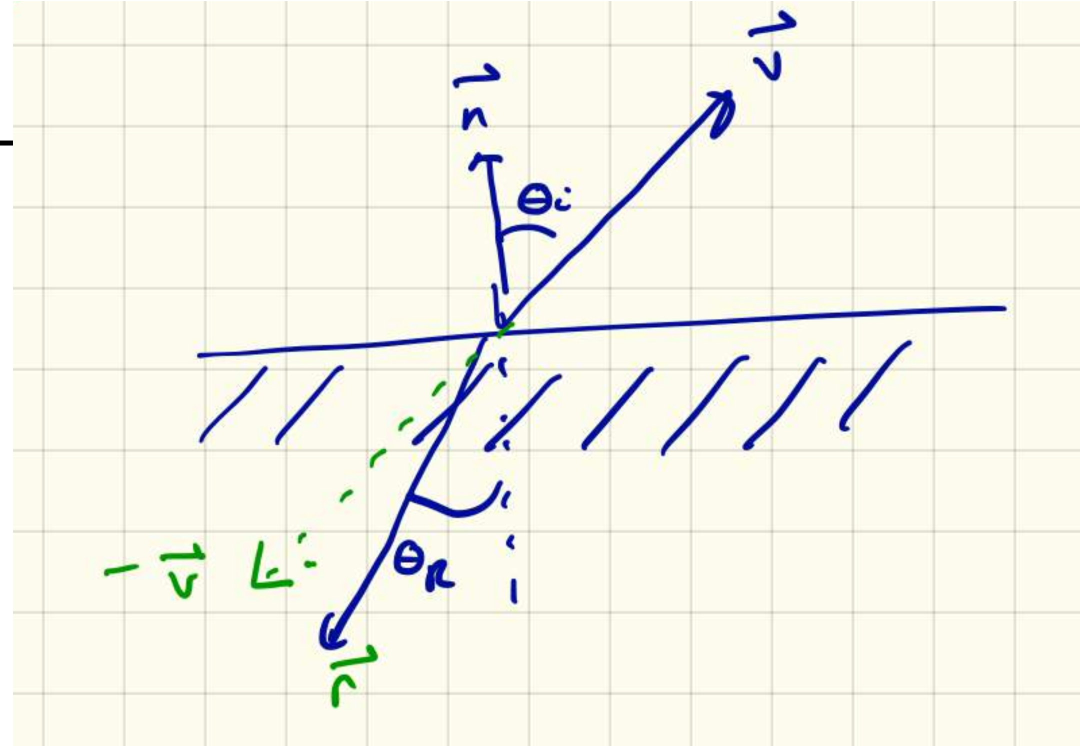
$$\vec{s}_\perp = (\vec{s} \cdot \vec{n})\vec{n}$$

$$\vec{s}_{||} = \vec{s} - (\vec{s} \cdot \vec{n})\vec{n}$$

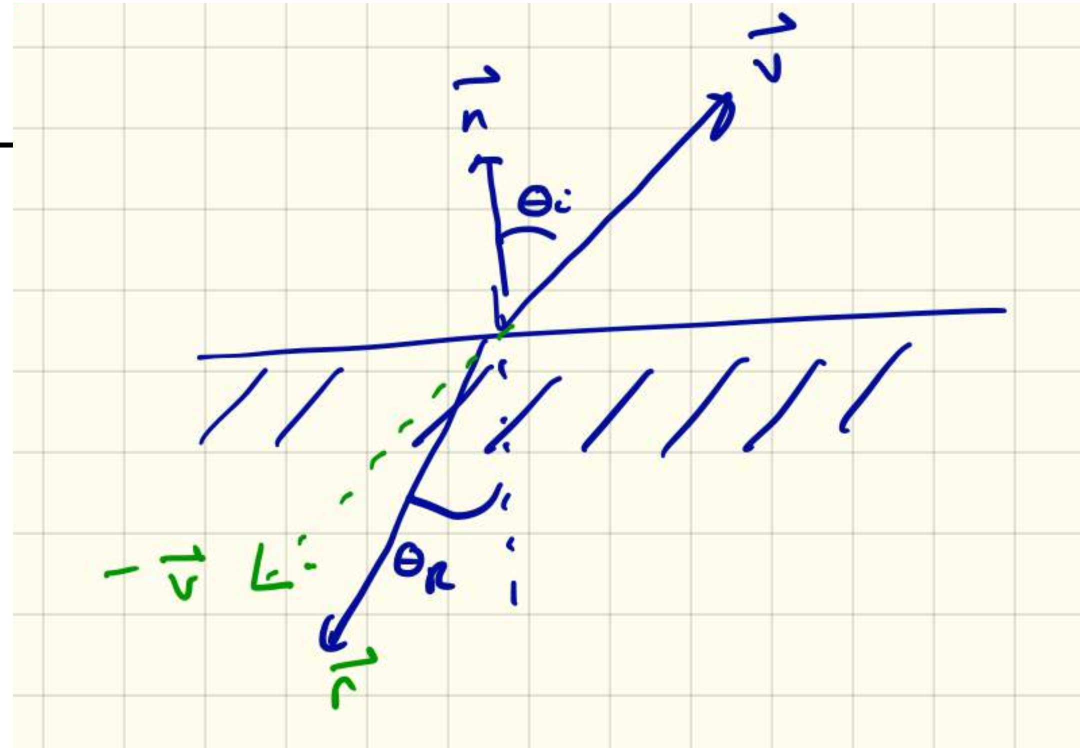
$$\sin\theta_1 = \frac{\|\vec{s}_{||}\|}{\|\vec{s}\|}$$

$$\sin\theta_2 = \frac{\|\vec{t}_{||}\|}{\|\vec{t}\|}$$

$$\|\vec{s}_{||}\| = \frac{c_1}{c_2}\|\vec{t}_{||}\|$$



$$\|\vec{s}_{\|}\| = \frac{c_1}{c_2}\|\vec{t}_{\|}\|$$



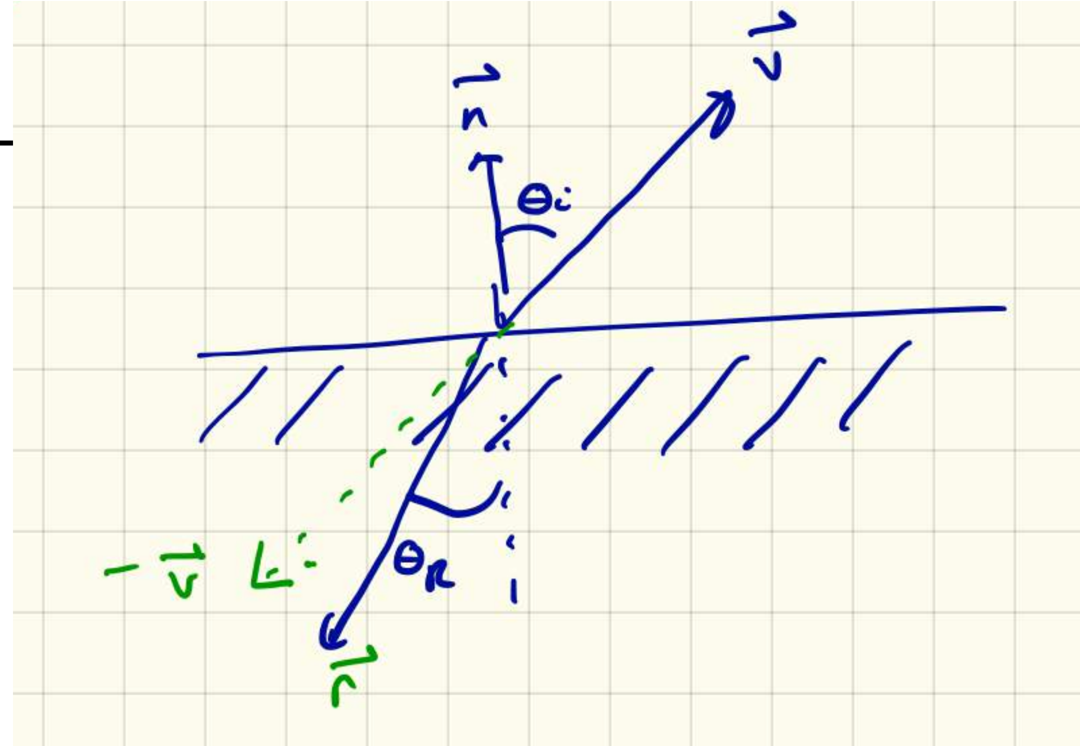
$$\vec{s}_{\|} = \vec{s} - (\vec{s}\cdot\vec{n})\vec{n}$$

$$\vec{t}_{\|} = -\frac{c2}{c1}\left(\vec{s} - \cos\theta_1\right)\vec{n}$$

$$\vec{t}_{\perp} = -\|\vec{t}_{\perp}\|\vec{n}$$

$$\vec{t}_{\perp} = -\sqrt{1 - \left(\frac{c2}{c1}\right)^2 \left(1 - \cos^2\theta_1\right)} \cdot \vec{n}$$

$$\vec{t} = \vec{t}_{\perp} + \vec{t}_{\|}$$



$$\vec{t} = \vec{t}_\perp + \vec{t}_\parallel$$

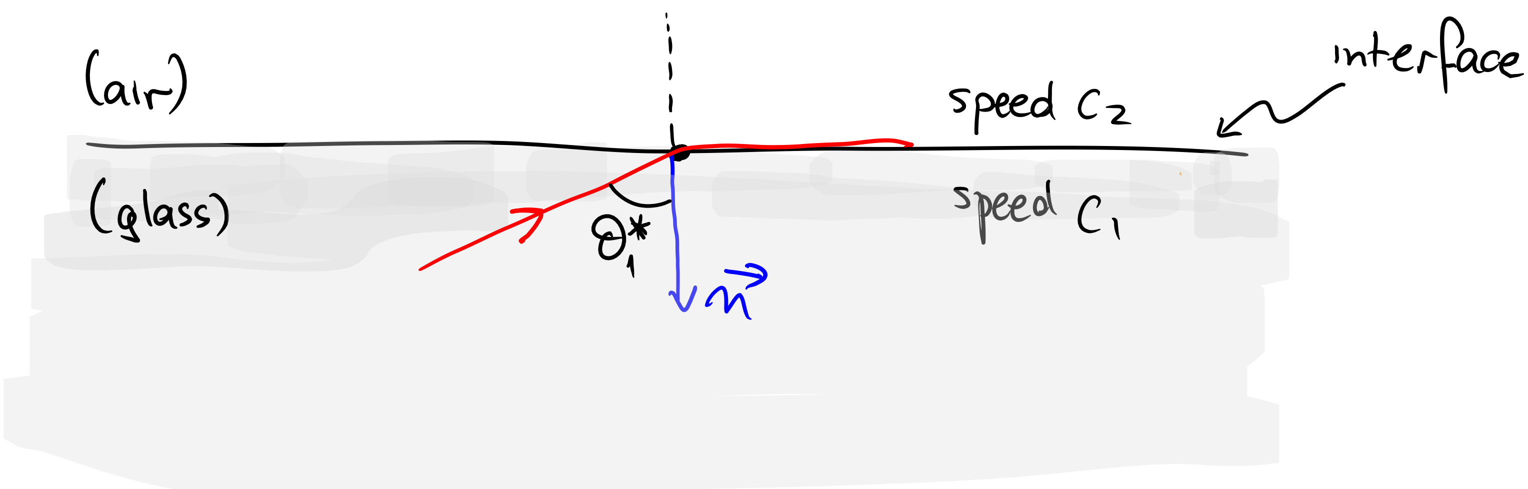
$$\vec{t} = -\frac{c2}{c1}\vec{s} + \left(\frac{c2}{c1}\cos\theta_1 - \cos\theta_2\right)\vec{n}$$

Q1: Define all the terms in this equation ?
Q2: Which terms are known and unknown during ray tracing ?
Q3: How do you compute the unknown terms ?

Geometry of Refraction: The Critical Angle

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$

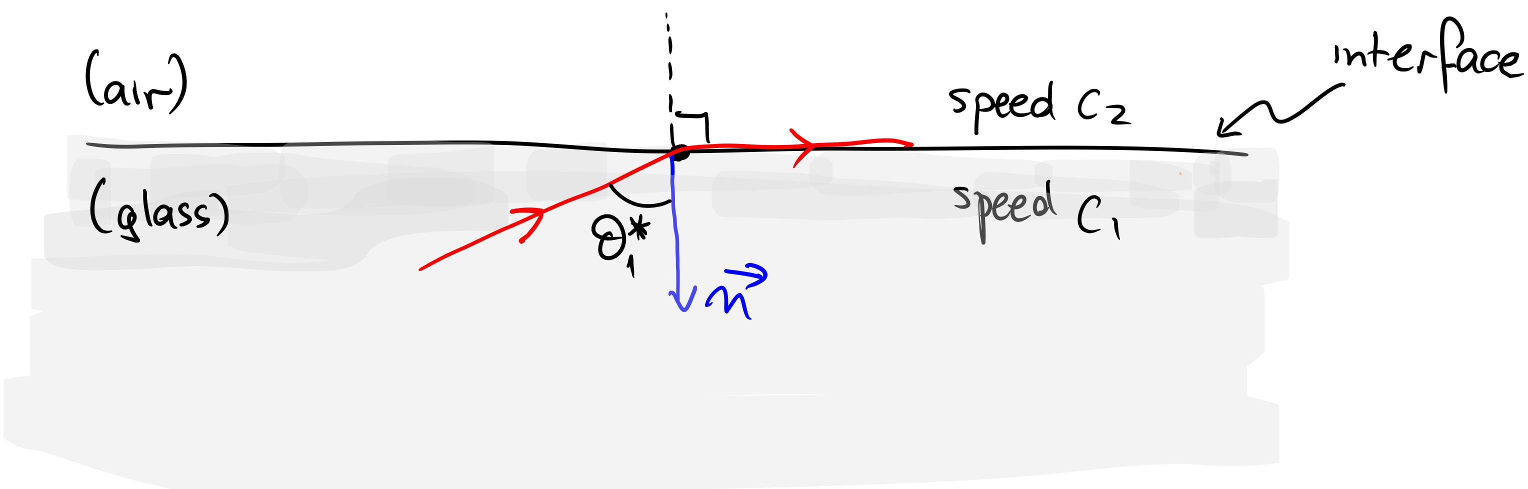


④ If $c_2 > c_1$ there is a critical angle above which no transmission occurs (⇒ have total internal reflection)

Total Internal Reflection

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$



④ Deriving the critical angle: from Snell's law,

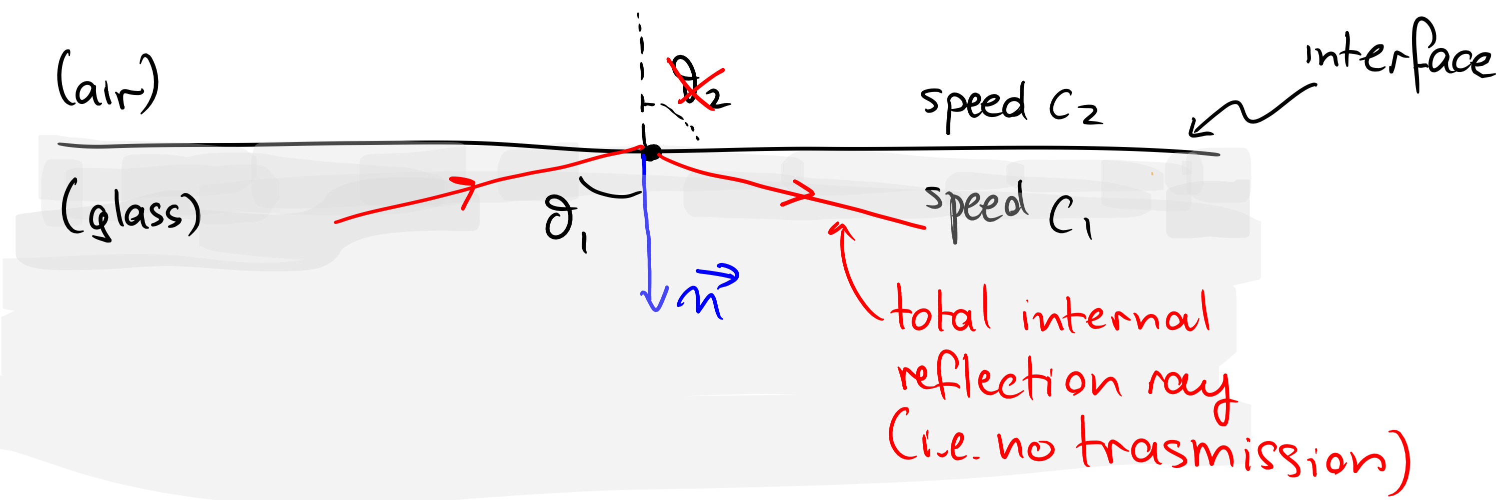
$$\cos\theta_2 = \sqrt{1-\left(\frac{c_2}{c_1}\right)^2\sin^2\theta_1}$$

at critical angle, $\theta_2 = \frac{\pi}{2} \Rightarrow \sin\theta_1^* = \frac{c_1}{c_2}$

Total Internal Reflection

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$

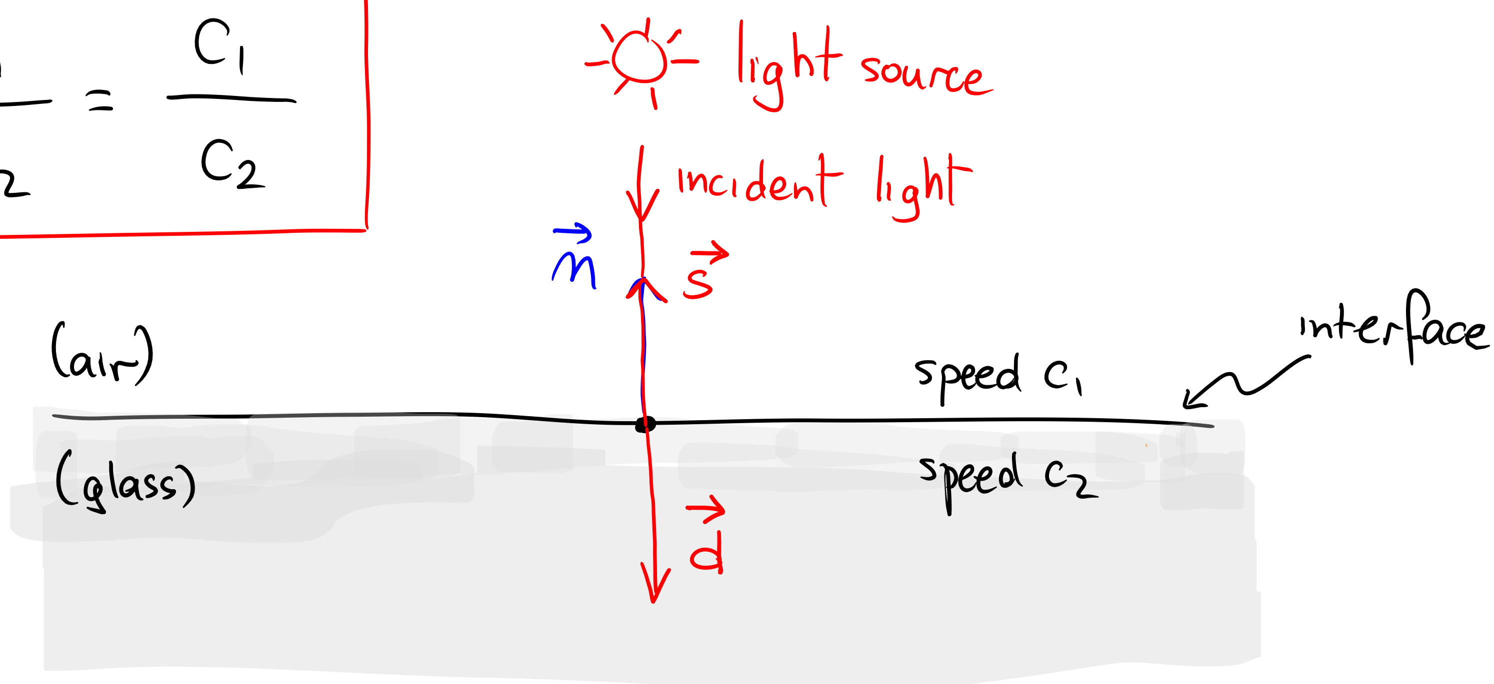


④ for $\theta_1 > \theta_1^*$, θ_2 is undefined

Geometry of Refraction: Normal Incidence

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$



⑤ If $\theta_1 = 0$, no bending occurs

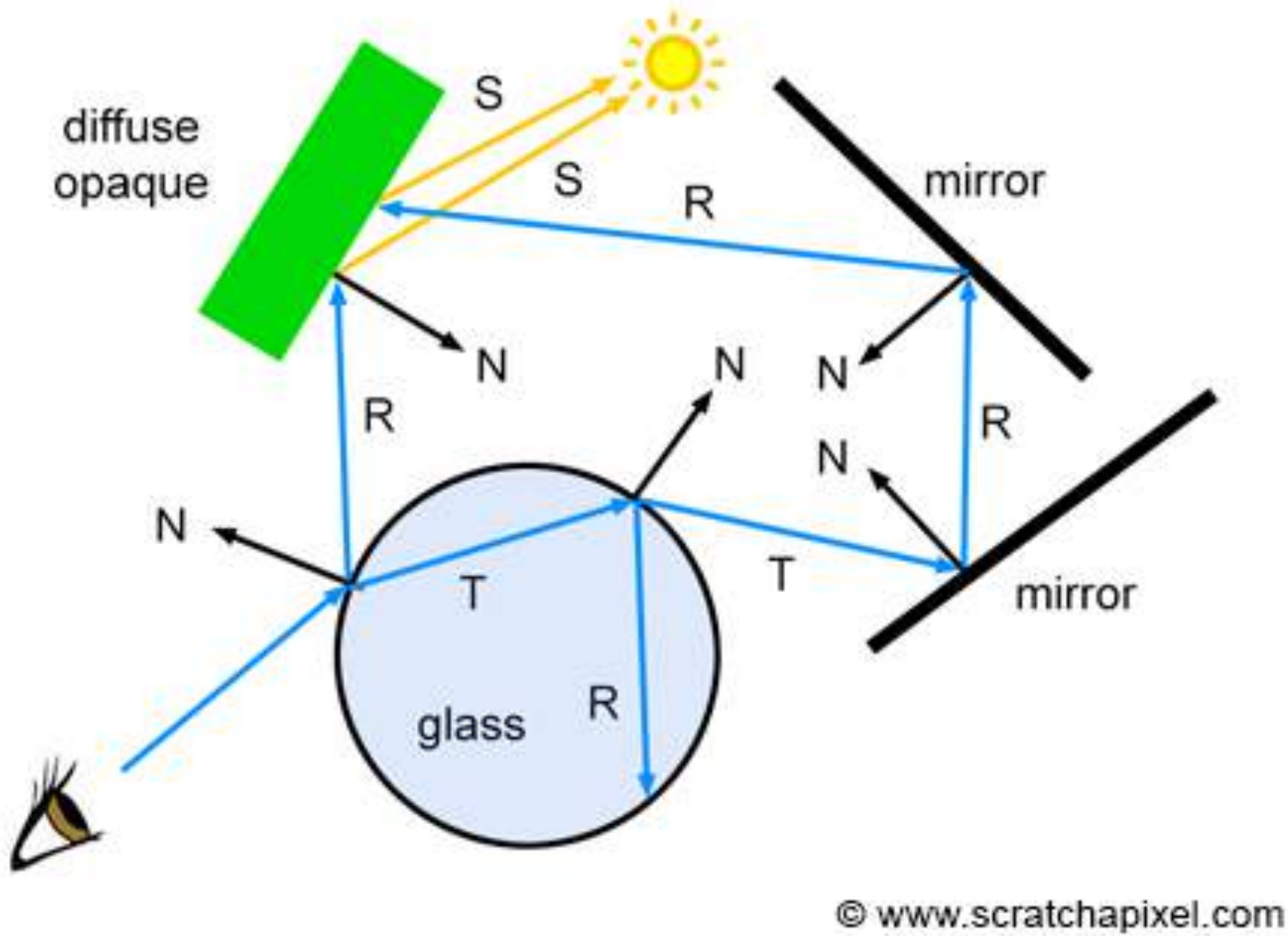
$$\vec{d} \text{ along } -\frac{c_2}{c_1}\vec{s} + \left[\frac{c_2}{c_1}\cos\theta_1 - \cos\theta_2\right]\vec{n}$$

Topic 12:

Less Basic Ray Tracing

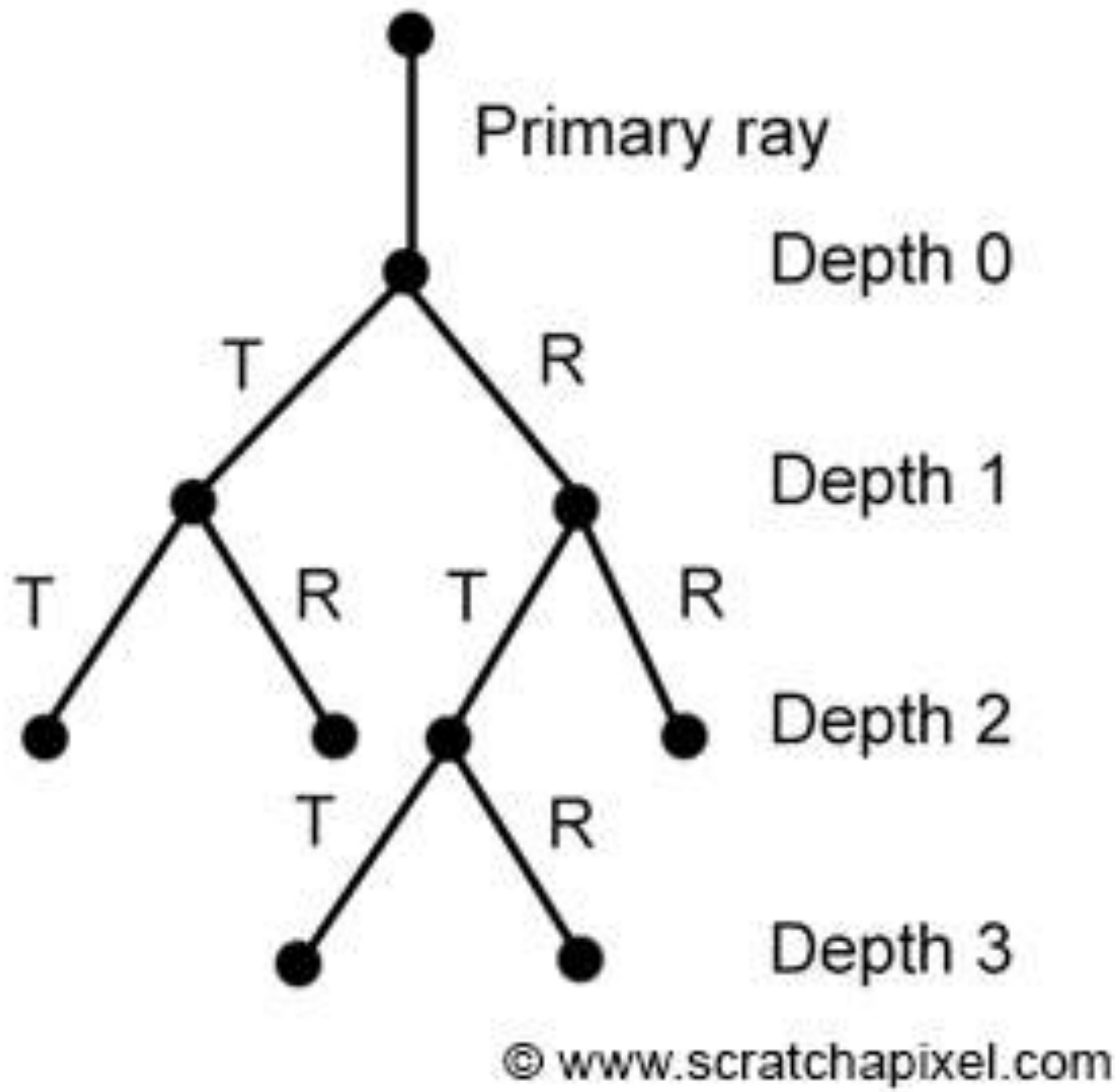
- Introduction to ray tracing
- Computing rays
- Computing intersections
  - ray-triangle
  - ray-polygon
  - ray-quadric
  - the scene signature
- Computing normals
- Evaluating shading model
- Spawning rays
- Incorporating transmission
  - refraction
  - ray-spawning & refraction

Ray Spawning

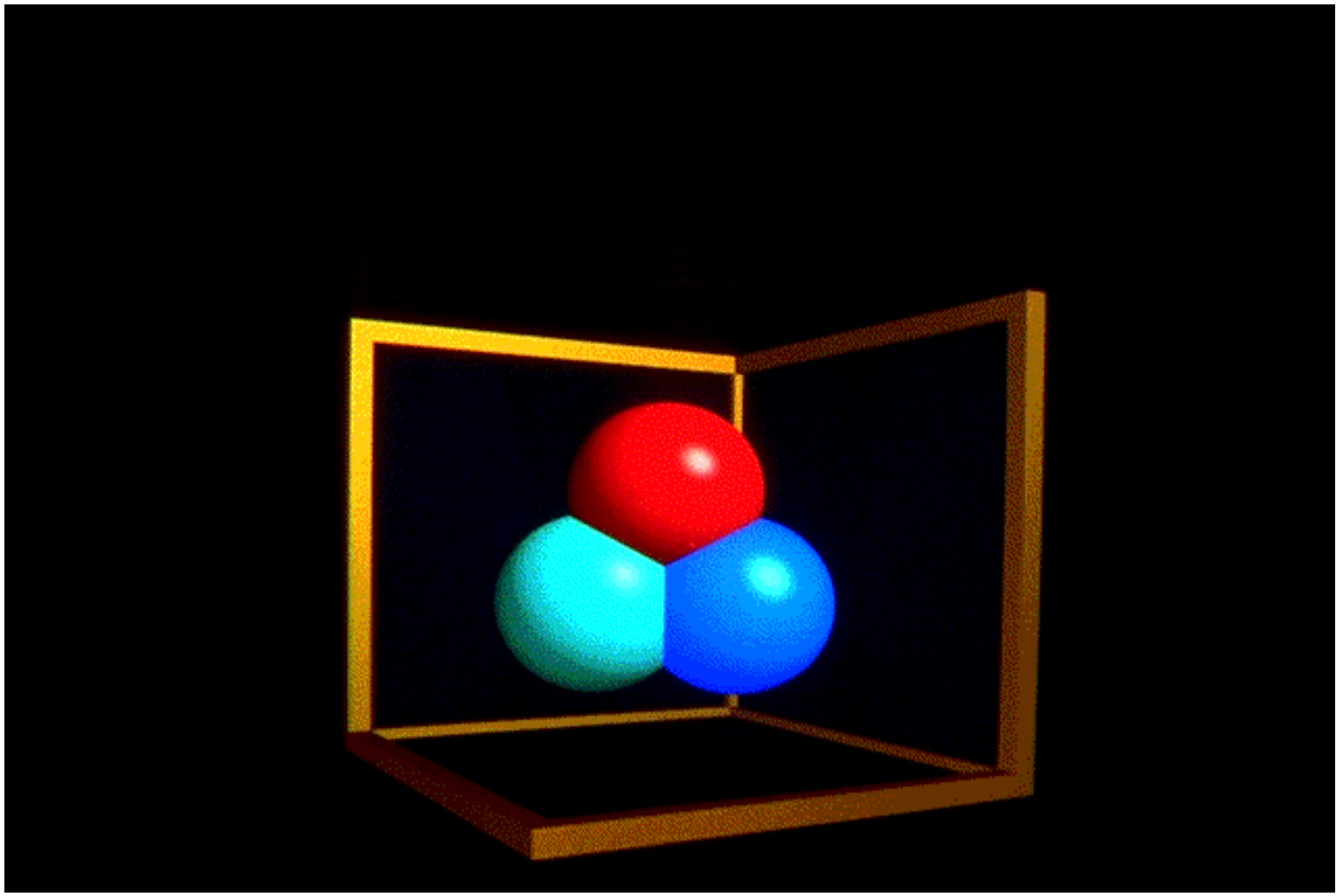


https://www.scratchapixel.com/lessons/3d-basic-rendering/ray-tracing-overview/light-transport-ray-tracing-whitted

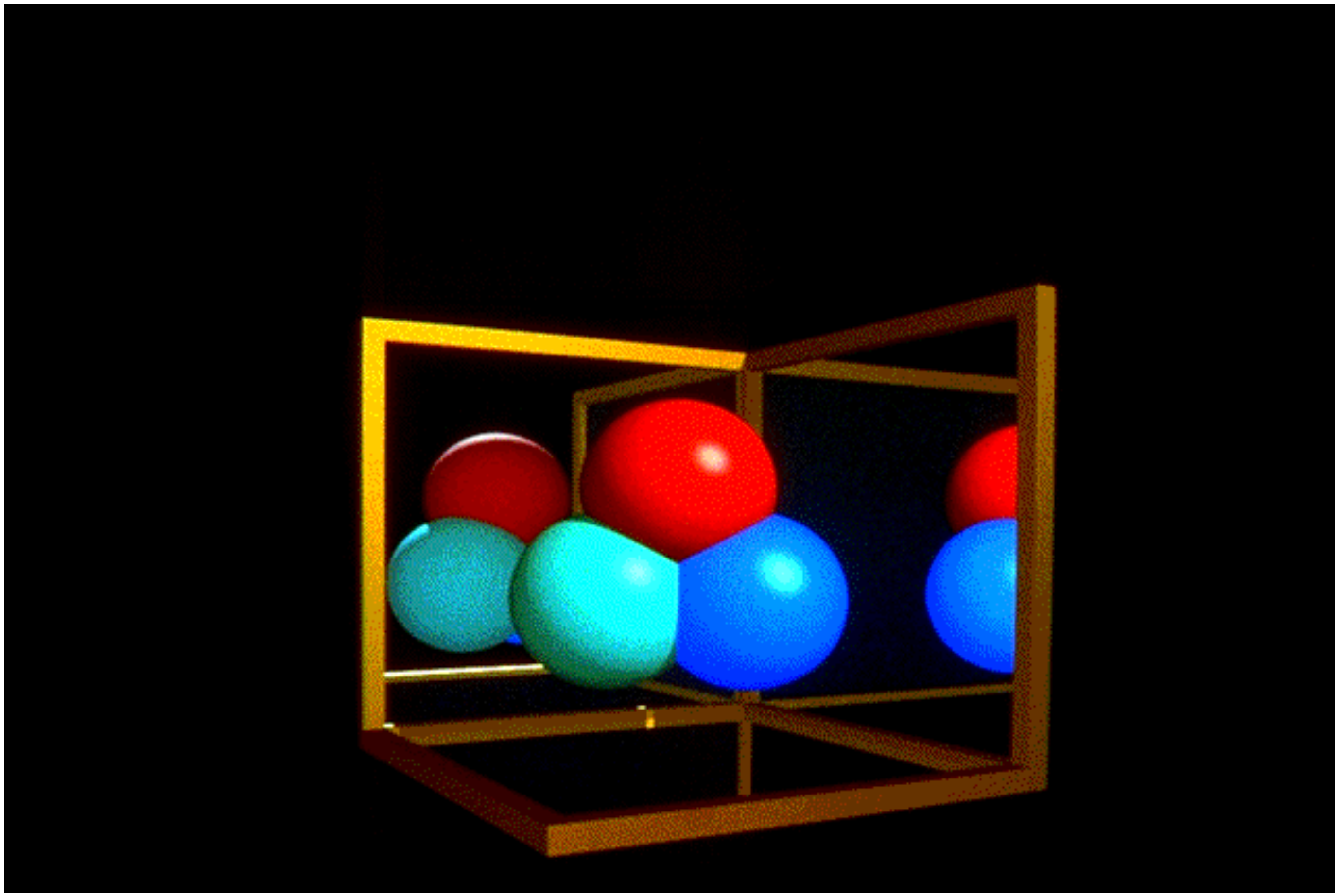
Ray Spawning: The Ray Tree



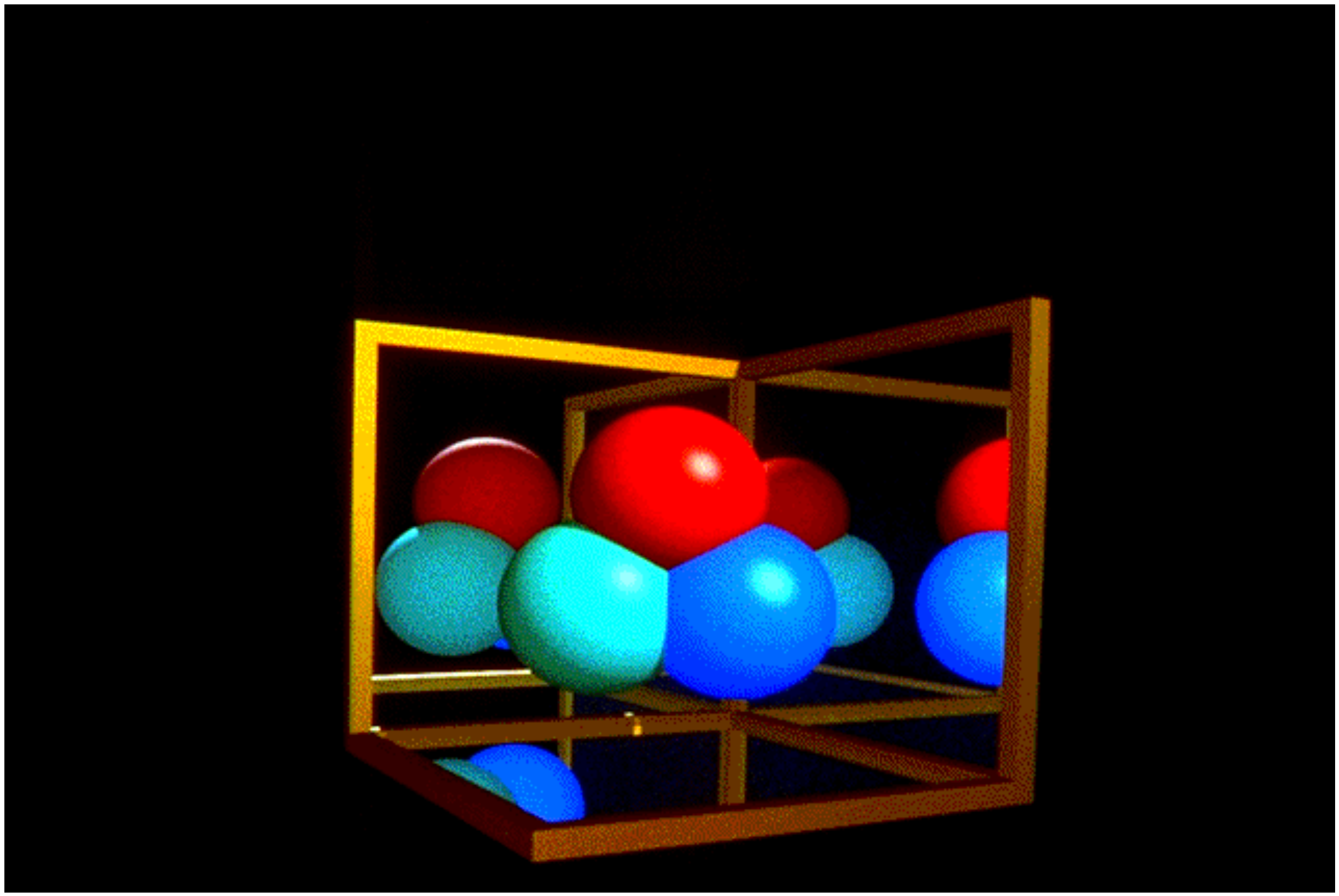
No reflection



Single reflection



Double reflection



Topic 12:

Less Basic Ray Tracing

- Introduction to ray tracing
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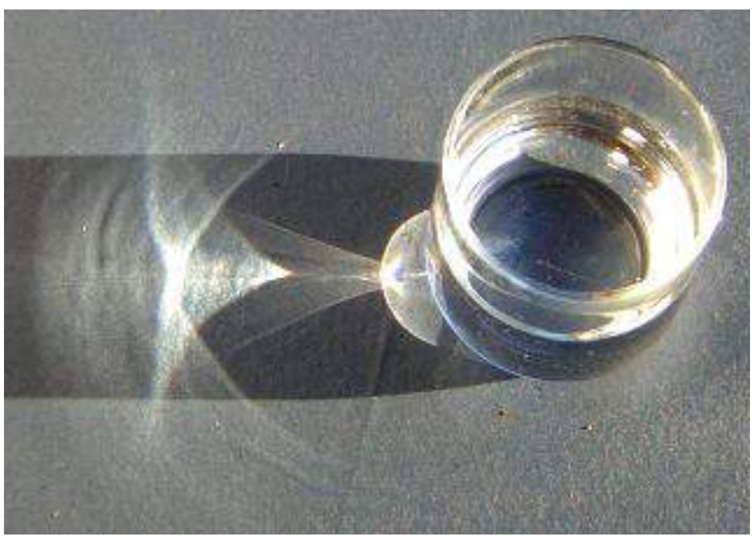
  - ray-spawning & refraction
  - Improvements

Ray Tracing Improvements: Caustics

Ray Tracing Improvements: Caustics

Reverse Direction Ray Tracing

- Trace from the light to the surfaces and then from the eye to the surfaces
- “shower” scene with light and then collect it
- “Where does light go?” vs “Where does light come from?”
- Good for caustics
- Transport E – S – S – S - D – S – S – S - L



Ray Tracing Improvements: Image Quality

Cone tracing

- Models some dispersion effects

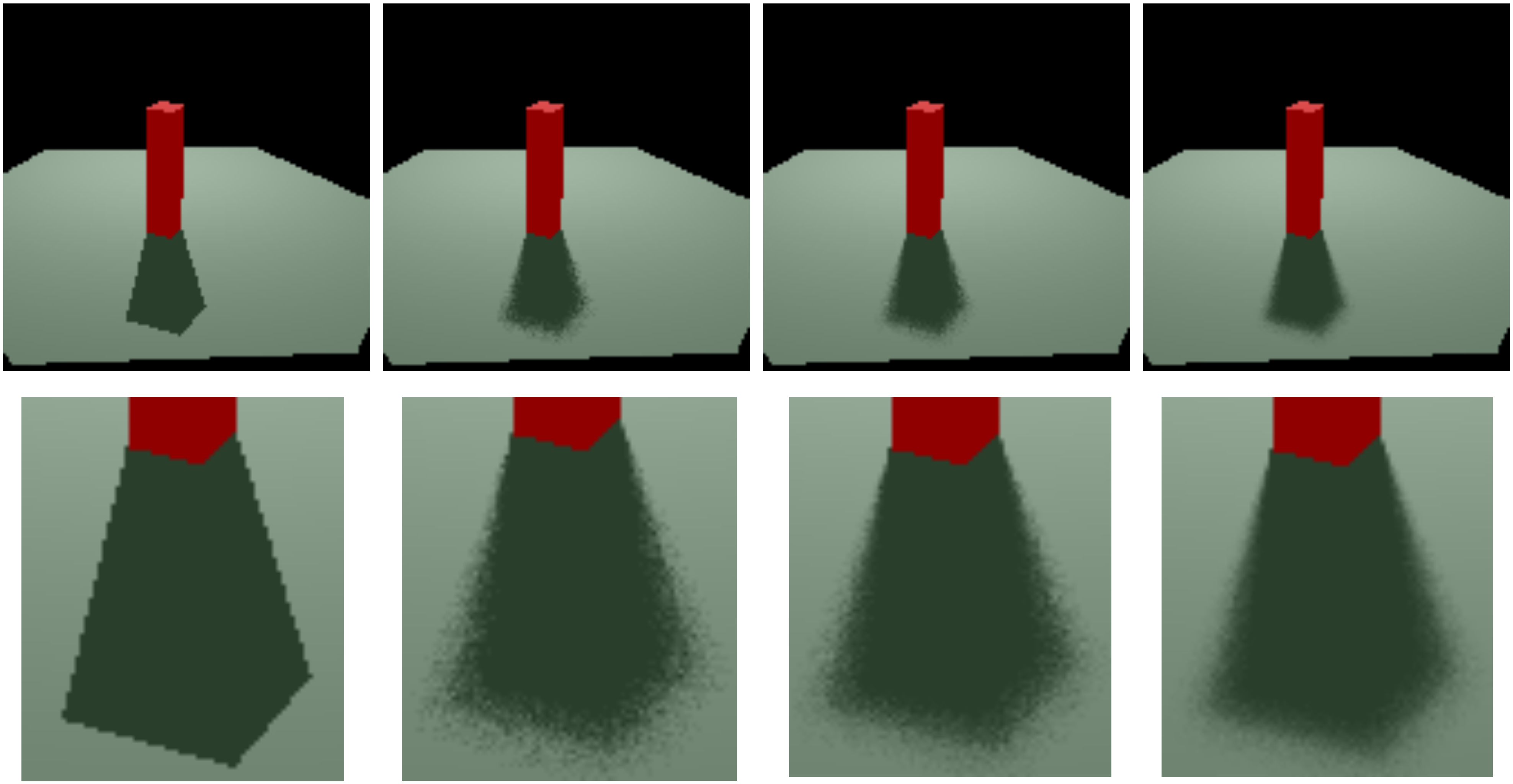
Distributed Ray Tracing

- Super sample each ray
- Blurred reflections, refractions
- Soft shadows
- Depth of field
- Motion blur

Stochastic Ray Tracing

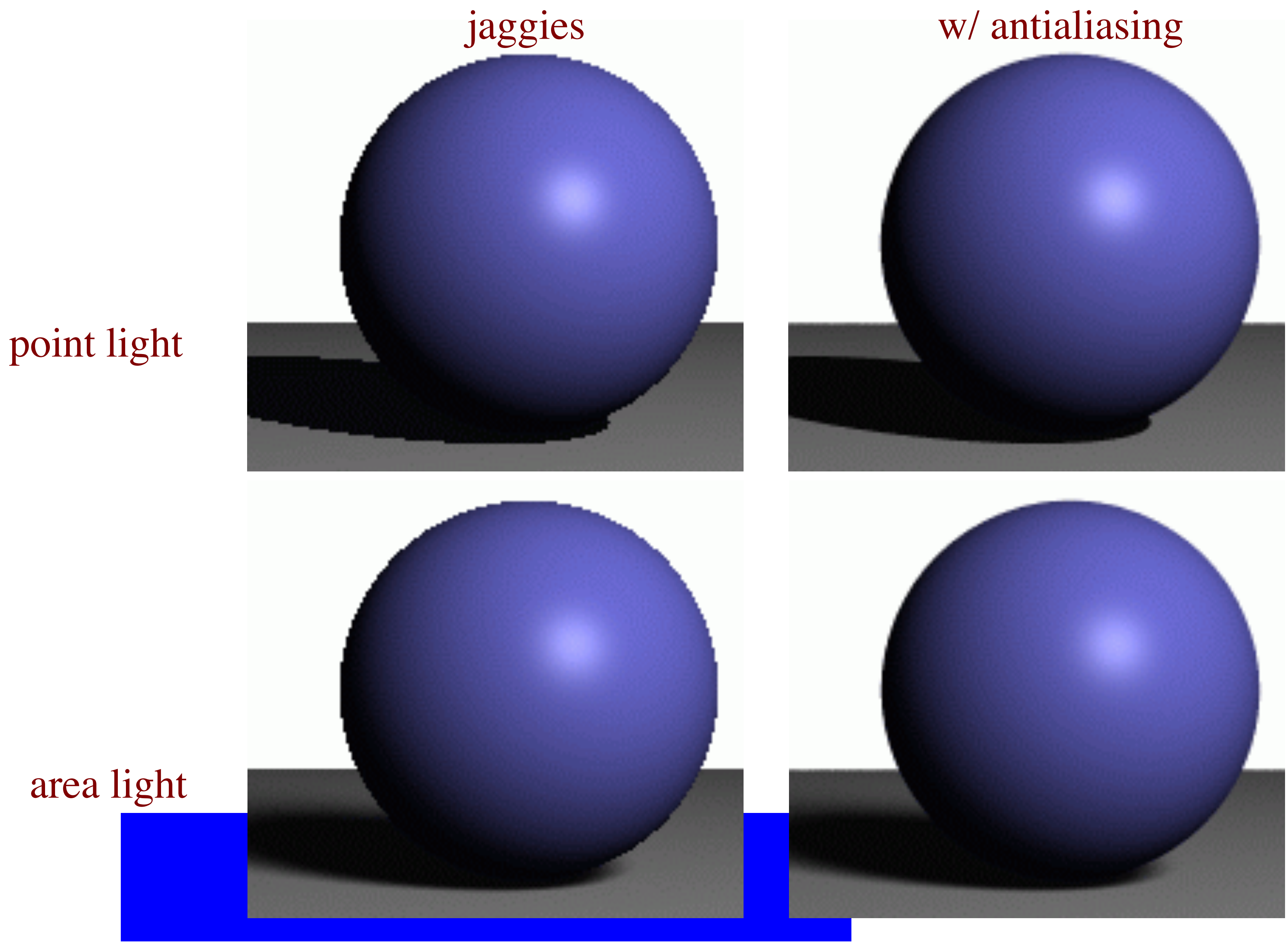
How many rays do you need?

1 ray/light | 10 ray/light | 20 ray/light | 50 ray/light



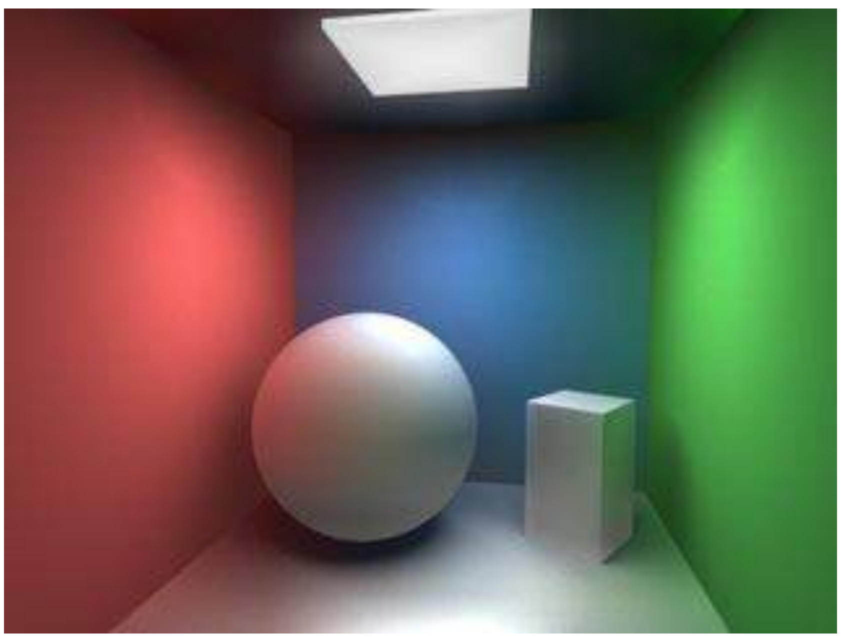
Images taken from http://web.cs.wpi.edu/~matt/courses/cs563/talks/dist_ray/dist.html

Antialiasing – Supersampling



Radiosity

- Diffuse interaction within a closed environment
- Theoretically sound
- View independent
- No specular interactions
- Color bleeding visual effects
- Transport E – D – D – D - L



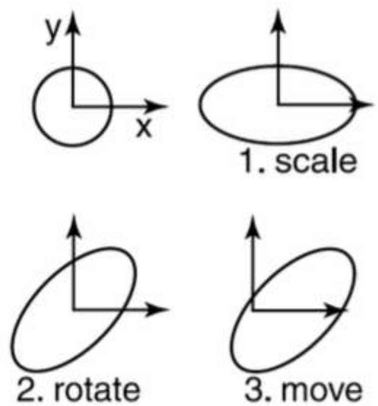
Topic 13:

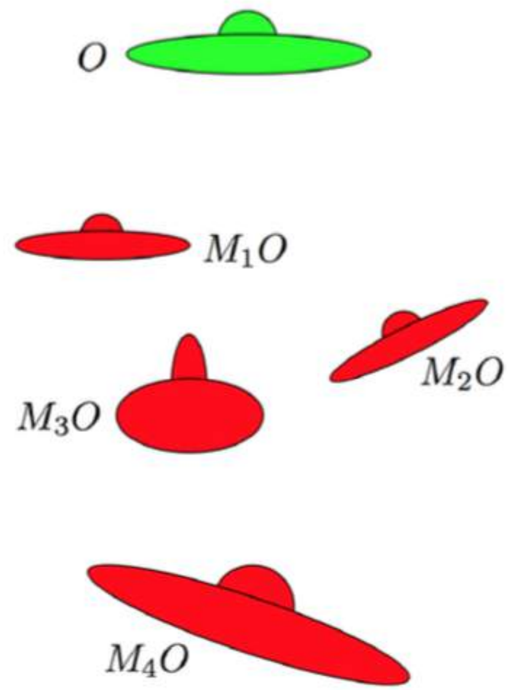
Instancing

Copying and transforming objects

Instancing is an elegant technique to place various transformed copies of an object in a scene.

Expl.: circle $\rightarrow$ elipse

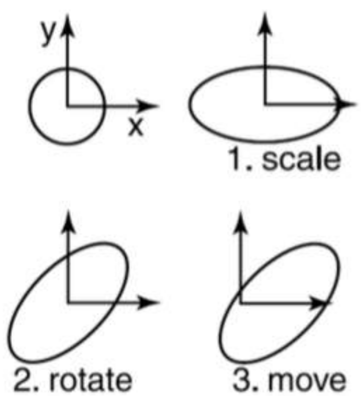


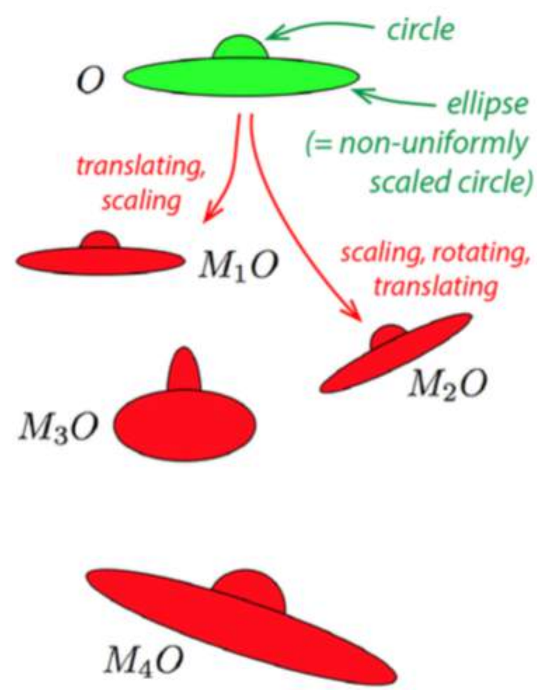


Copying and transforming objects

Instancing is an elegant technique to place various transformed copies of an object in a scene.

Expl.: circle $\rightarrow$ elipse



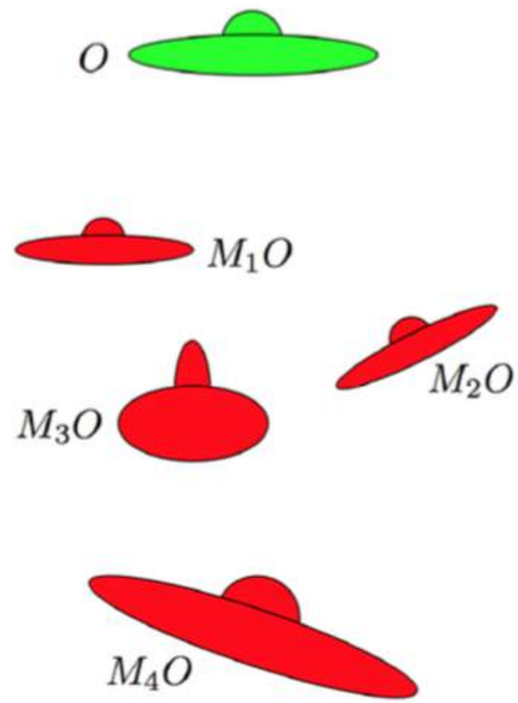


Copying and transforming objects

Instead of making actual copies, we simply store a reference to a base object, together with a transformation matrix.

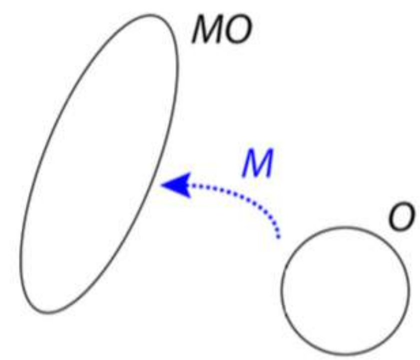
That can save us lots of storage.

Hmm, but how do we compute the intersection of a ray with a randomly rotated elipse?



Ray-instance intersection

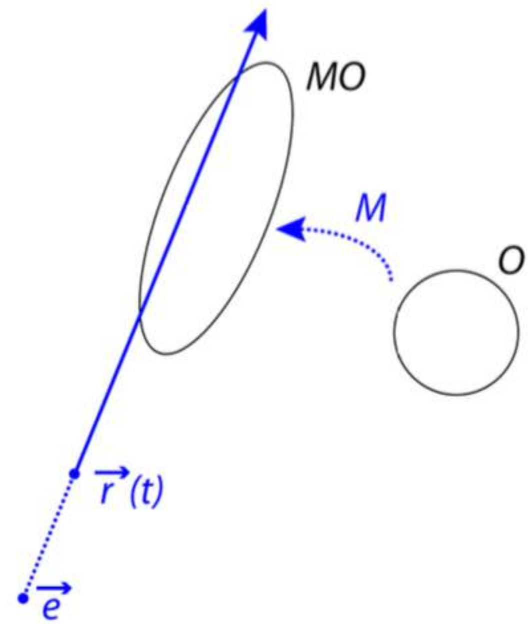
Assume an object O that is used to create an object MO via instancing.



Ray-instance intersection

Now, we want to create the intersection of MO with the ray $\vec{r}(t)$, which in turn is defined by the line

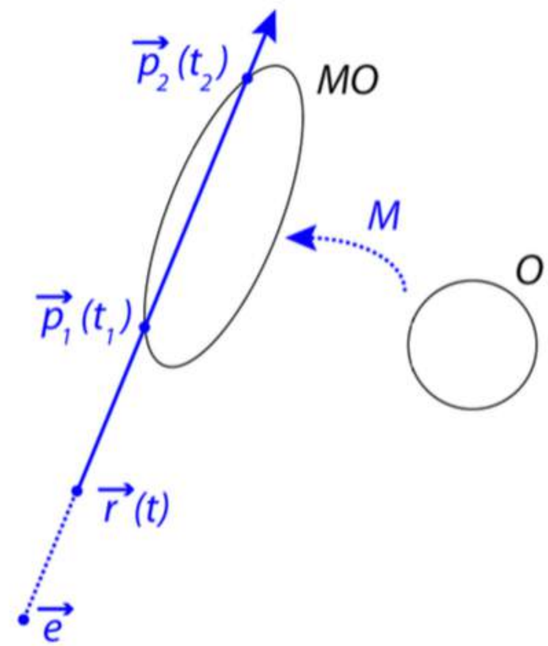
$$\vec{l}(t) = \vec{e} + t\vec{d}.$$



$\vec{l}(t) = \vec{e} + t\,\vec{d}$

Ray-instance intersection

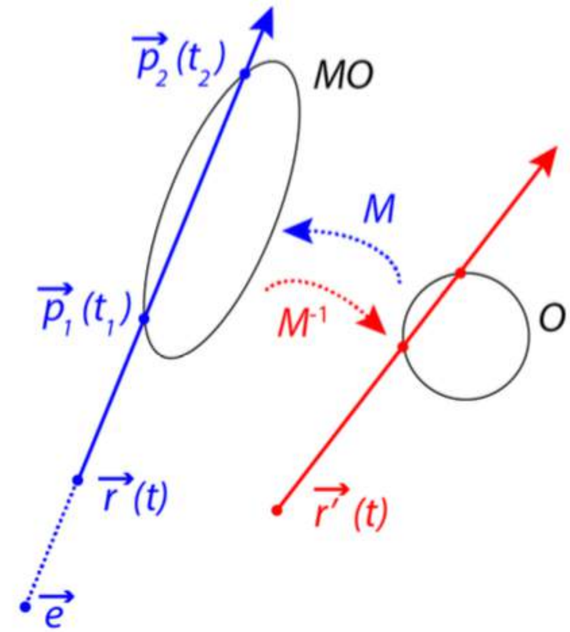
Fortunately, such complicated intersection tests (e.g. ray/ellipsoid) can often be replaced by much simpler tests (e.g. ray/sphere).



$\vec{l}(t) = \vec{e} + t\,\vec{d}$

Ray-instance intersection

To determine the intersections $\vec{p_i}$ of a ray $\vec{r}$ with the instance MO, we first compute the intersections $\vec{p_i'}$ of the inverse transformed ray $M^{-1}\vec{r}$ and the original object O.



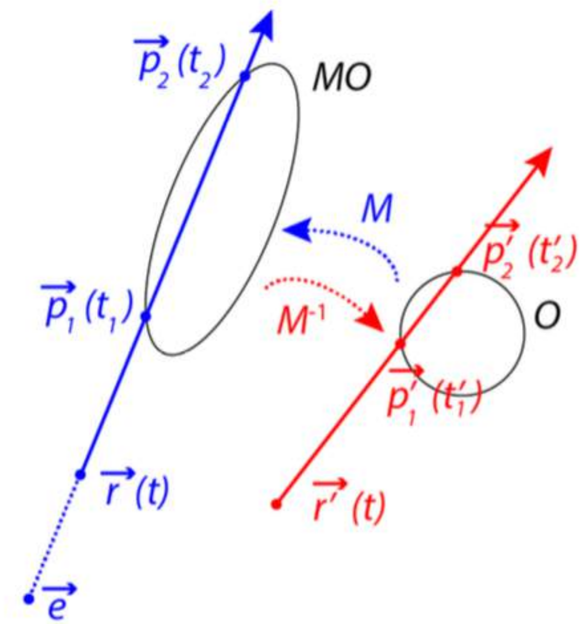
$$\vec{l}(t) = \vec{e} + t\,\vec{d}$$

Ray-instance intersection

The points $\vec{p_i}$ are then simply

$$M\vec{p_i'} \quad \text{or} \quad \vec{l}(t_i')$$

because the linear transformation preserves relative distances along the line.

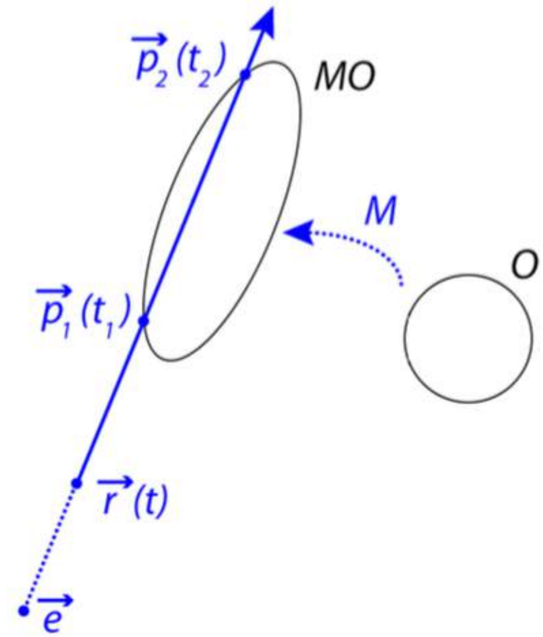


$\vec{l}(t) = \vec{e} + t\,\vec{d}$

Ray-instance intersection

Two pitfalls:

- The direction vector of the ray should *not* be normalized
- Surface normals transform differently!
  $\rightarrow$ use $(M^{-1})^T$ instead of M for normals



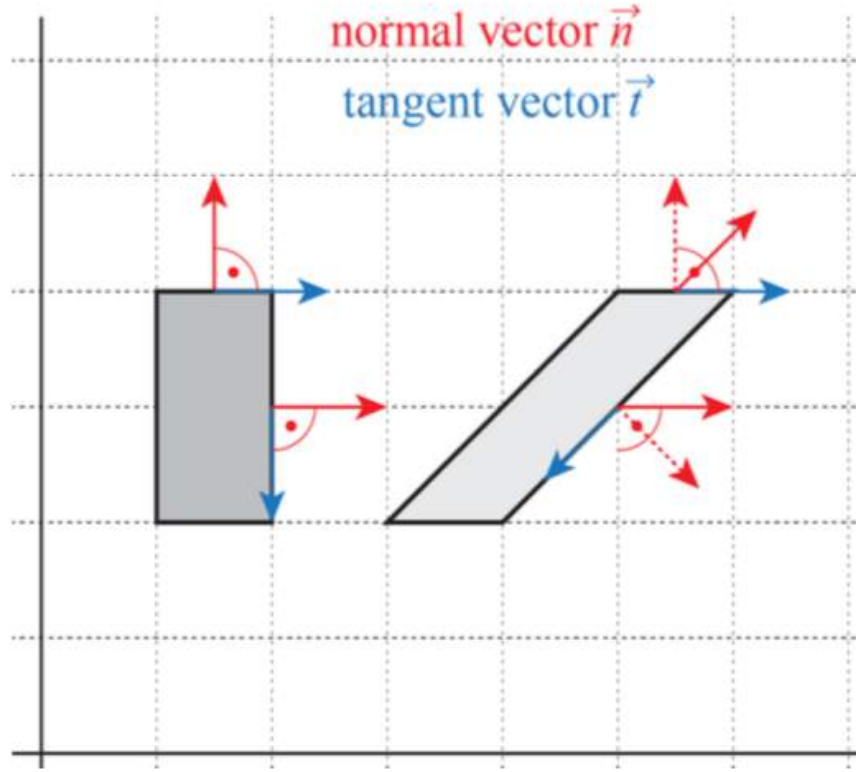
$\vec{l}(t) = \vec{e} + t\,\vec{d}$

Transforming normal vectors

Unfortunately, normal vectors are not always transformed properly.

E.g. look at shearing, where tangent vectors are correctly transformed but normal vectors not.

To transform a normal vector $\vec{n}$ correctly under a given linear transformation A, we have to apply the matrix $(A^{-1})^T$. Why?



Transforming normal vectors

We know that tangent vectors are tranformed correctly: $A\vec{t} = \vec{t_A}$.

But this is not necessarily true for normal vectors: $A\vec{n} \neq \vec{n_A}$.

Goal: find matrix N_A that transforms $\vec{n}$ correctly, i.e. $N_A\vec{n} = \vec{n_N}$ where $\vec{n_N}$ is the correct normal vector of the transformed surface.

Because our original normal vector $\vec{n}^T$ is perpendicular to the original tangent vector $\vec{t}$, we know that:

$$\vec{n}^T\vec{t} = 0.$$

This is the same as

$$\vec{n}^T I\vec{t} = 0$$

which is is the same as

$$\vec{n}^T A^{-1} A\vec{t} = 0$$

Transforming normal vectors

Because $A\vec{t} = \vec{t_A}$ is our correctly transformed tangent vector, we have

$$\vec{n}^T A^{-1} \vec{t_A} = 0$$

Because their scaler product is 0, $\vec{n}^T A^{-1}$ must be orthogonal to it. So, the vector we are looking for must be

$$\vec{n}_N^T = \vec{n}^T A^{-1}.$$

Because of how matrix multiplication is defined, this is a transposed vector. But we can rewrite this to

$$\vec{n}_N = (\vec{n}^T A^{-1})^T.$$

And if you remember that $(AB)^T = B^T A^T$, we get

$$\vec{n}_N = (A^{-1})^T \vec{n}$$