CSC 418/2504 Computer Graphics, Winter 2018 Assignment 1 (10% of course grade)

Due 11:59pm on Wed., Jan. 31, 2018.

Part A [45 marks in total]

Below are 4 exercises covering different topics from the weeks of class upto reading week. They require thought, so you are advised to consult the relevant sections of the textbook, the online lecture notes and slides, and your notes from class well in advance of the due date. Your proofs and derivations should be clearly written, mathematically correct, and concise.

All questions require showing the steps toward the solution, and marks will be subtracted if this is not the case. Even if you cannot answer a question completely, it is very important that you show your (partial) answers and your reasoning. Otherwise your TA will **not** be able to award you partial marks. Both Part A and Part B must be done individually and electronically submitted. Part A must be in PDF format, by scanning your handwritten solution or by using LaTeX/word to typeset it.

1. Curves [15 marks]

The dress code for the wooden monkey awards includes a bow tie, described in 2D with the following parametric form: x(t) = 2*sin(t), y(t) = 5*sin(t)*cos(t), $0 \le t \le 2\pi$.

Can you write the above function in its implicit form [3 marks]. Find the tangent vector [2 marks] and a normal vector [2 marks] to the curve as a function of *t*.

Is the curve symmetric around the X-axis? Y-axis? (proof/counterexample) [2 marks]. What is the enclosed area of the bow-tie [3 marks]?

How can one piecewise linearly approximate the perimeter of the bowtie [3 marks]?

2. Transformations [10 marks]

Two transformations T1 and T2 commute when T1 * T2 = T2 * T1. A point *p* is a fixed point of a transformation *T* if and only if Tp = p. For each pair of transformations below, specify whether or not they commute in general. Moreover, if you conclude that they commute, provide a proof, and if you claim the converse, provide a counterexample as proof [2.5 marks for each part]. (a) translation and translation.

- (b) translation and rotation.
- (c) scaling and rotation, having different fixed points.
- (d) scaling and scaling, having the same fixed point.
- 3. Homography [10 marks]

Points (1,0); (0,1); (1,1); (0,0) map to points (6,2); (6,3); (7,3); (7,2) by an Affine transformation. [8 marks] Derive the Affine transformation. Simply write the steps needed to find it. [2 marks] Where does the point (2,5) get mapped to under this transformation?

4. Polygons [10 marks]

Given an arbitrary, non-degenerate 2D triangle with vertices v0, v1, and v2, write a procedure for determining if a point q is inside/outside the triangle [4 marks], or on an edge of the triangle [2 marks]. The procedure can be described or in pseudocode, as long as the steps are clear. How can one compute the area of a triangle and its centroid (also its center of mass) [4 marks]?