

Dynamically Coupled Particle Systems  
for Geometric Modeling, Reconstruction, and Animation

by

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A thesis submitted in conformity with the requirements  
for the degree of Doctor of Philosophy  
Graduate Department of Computer Science  
University of Toronto

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## Abstract

This dissertation presents a new technique, based on dynamically coupled particle systems, for creating and manipulating complex three dimensional shapes in a fluid like manner. The most novel feature of this approach to shape representation is the use of self organizing primitive elements. In the simplest case, these primitive elements, or particles, each possess state variables of position and mass, and the system of elements interact through pairwise potential energy functions. More complex systems include additional state variables combined with simple heuristics to create application specific behavior. The ability of these systems to self organize provides a representation technique which exhibits dynamically changing structure, an attribute not found in popular spline and polygonally based representations. To illustrate the usefulness of this approach it is applied to the following problems: free form shape modeling, computer assisted animation, and surface reconstruction. For free-form modeling the approach supports smoothness constraints similar to those inherent in the deformation energies of popular, elastic surface models. Unlike spline patches or parameterized surface models, the model does not attempt to enforce analytical continuity conditions, such as tangent or curvature continuity over the surface. Applied to computer assisted animation the approach computes the movement and deformation of models mimicking, at a rudimentary level, the physical behavior of flexible solids and fluids. Applied to surface reconstruction, these systems can infer surface structure from sparse data sets, without a prior knowledge of the surface structure or the topological genus. In summary, dynamically coupled particle systems provide an useful alternative to traditional shape representation and manipulation techniques.



## Acknowledgments

I thank my advisors, Demetri Terzopoulos and Richard Szeliski, for their continued support and advice. I thank Bill Buxton, Eugene Fiume, Michiel van de Panne, and James Stewart, for participating as members of my Ph. D. committee and for their insights and advice. I thank Gavin Miller for fulfilling the role of external appraiser. I also thank the many members of the Dynamic Graphics Project for their support, friendship, and inspiration.

I thank the University of Toronto, Department of Computer Science and the Digital Equipment Corporation, Cambridge Research Lab for the financial support and the facilities used to pursue this research.

I thank my family and my friends who have given me moral support, counsel, and friendship. I thank Bob, Diane, Fabio, Felix, Goofina, Huleo, Jason, Jay, Kathleen, Litsa, Rakesh, Rania, Ray, Rebecca, the Sanchez family, Spider Dave, Steve, Tasso, and William. And in particular, I thank Jos, Melanie, Pam, Rakesh, Rod, and Sylvia.



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